03-04-25 MTH 161-004N

2.3 Quadratic Functions
2.3.1 Exercises
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2.4 Inequalities with Absolute Value and Quadratic Functions
2.4.1 Exercises
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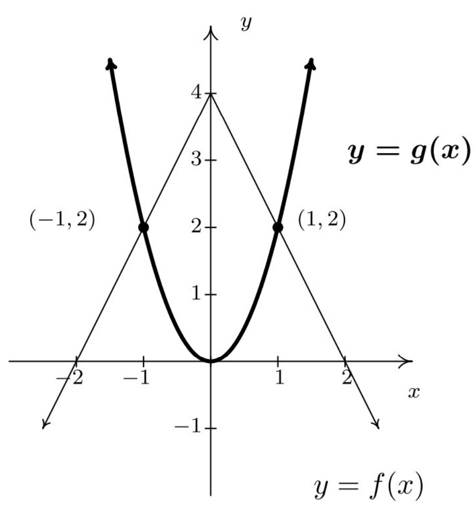
memorize

Graphical Interpretation of Equations and Inequalities

Suppose f and g are functions.

- The solutions to $f(x) = g(x)$ are the x values where the graphs of $y = f(x)$ and $y = g(x)$ intersect.
- The solution to $f(x) < g(x)$ is the set of x values where the graph of $y = f(x)$ is *below* the graph of $y = g(x)$.
- The solution to $f(x) > g(x)$ is the set of x values where the graph of $y = f(x)$ *above* the graph of $y = g(x)$.

Example 2.4.2. The graphs of f and g are below. (The graph of $y = g(x)$ is bolded.) Use these graphs to answer the following questions.



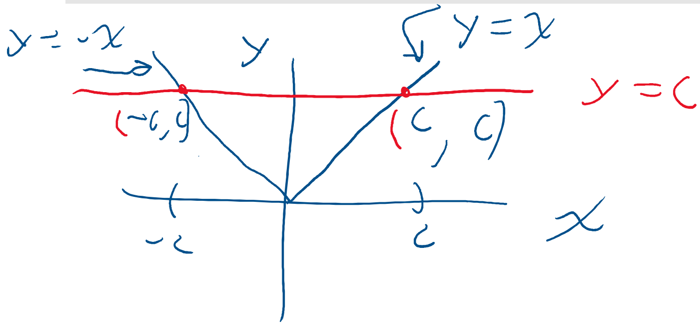
1. Solve $f(x) = g(x)$.
   $x = \pm 1$
2. Solve $f(x) < g(x)$.
   $(-\infty, -1) \cup (1, \infty)$
3. Solve $f(x) \geq g(x)$.
   $[-1, 1]$

Memorize

Theorem 2.4. Inequalities Involving the Absolute Value: Let c be a real number.

- For $c > 0$, $|x| < c$ is equivalent to $-c < x < c$.
- For $c > 0$, $|x| \leq c$ is equivalent to $-c \leq x \leq c$.

- For $c > 0$, $|x| < c$ is equivalent to $-c < x < c$.
- For $c > 0$, $|x| \le c$ is equivalent to $-c \le x \le c$.
- For $c \le 0$, $|x| < c$ has no solution, and for $c < 0$, $|x| \le c$ has no solution.
- For $c \ge 0$, $|x| > c$ is equivalent to $x < -c$ or $x > c$.
- For $c \ge 0$, $|x| \ge c$ is equivalent to $x \le -c$ or $x \ge c$.
- For $c < 0$, $|x| > c$ and $|x| \ge c$ are true for all real numbers.



Numeric test value method

Steps for Solving a Quadratic Inequality

1. Rewrite the inequality, if necessary, as a quadratic function $f(x)$ on one side of the inequality and 0 on the other.
2. Find the zeros of f and place them on the number line with the number 0 above them.
3. Choose a real number, called a **test value**, in each of the intervals determined in step 2.
4. Determine the sign of $f(x)$ for each test value in step 3, and write that sign above the corresponding interval.
5. Choose the intervals which correspond to the correct sign to solve the inequality.

Solve $f(x) \le 4$

with $f(x) = x^2 - 2x - 3$

$x^2 - 2x - 3 \le 4$

Let $g(x) = x^2 - 2x - 7$

solve $g(x) \le 0$

1. $x^2 - 2x - 3 - 4 \le 0$

$x^2 - 2x - 7 \le 0$

2. solve $x^2 - 2x - 7 = 0$

$$x = \frac{2 \pm \sqrt{4 - (4)(1)(-7)}}{2}$$

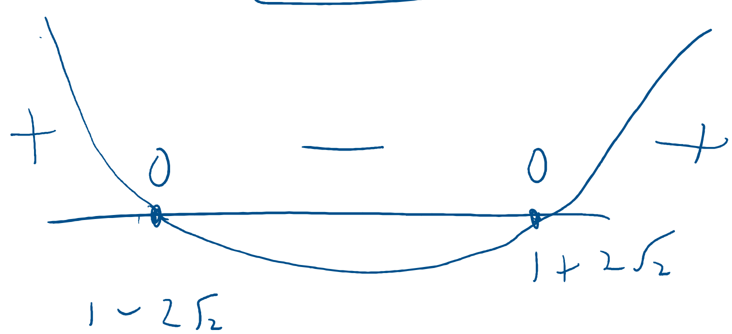
$$x = \frac{2 \pm \sqrt{4 + 28}}{2}$$

$$x = \frac{2 \pm \sqrt{32}}{2}$$

$$x = \frac{2 \pm \sqrt{16 \cdot 2}}{2}$$

$$x = \frac{2 \pm 4\sqrt{2}}{2}$$

$$\boxed{x = 1 \pm 2\sqrt{2}}$$



$g(x) = x^2 - 2x - 7$

1-sqrt(2)=-0.414213562373095
1+sqrt(2)=2.414213562373095

$$(-\infty, 1-2\sqrt{2}), \ (1-2\sqrt{2}, 1+2\sqrt{2}), \ (1+2\sqrt{2}, \infty)$$

$$g(-4) = (-4)^2 - 2(-4) - 7 = 16 + 8 - 7 = 15 > 0$$

$$g(1) = {}^2 - 2(1) - 7 = 1 - 9 = -8 < 0$$

$$g(4) = 4^2 - 2(4) - 7 = 16 - 15 = 1 > 0$$

$$\boxed{\text{solution } [1 - 2\sqrt{2}, 1 + 2\sqrt{2}]}$$

algebra / geometry

Solve $f(x) > 0$

$f(x) = (x - 5)(x + 7) \quad - \ x^2 + \text{other stuff}$

Solve $f(x) > 0$

for $f(x) = (x-5)(x+7) = x^2 + \text{other stuff}$

positive coeff

$\therefore$ parabola open up

$(-\infty, -7) \cup (5, \infty)$

$f(x) = -4x^2 + 10x - 5 > 0$

solve $f(x) = 0$

Find exact zeros, then use calculator graph

$-4x^2 + 10x - 5 = 0$

$$x = \frac{-10 \pm \sqrt{100 - (-4)(-5)(4)}}{-8}$$

$$x = \frac{-10 \pm \sqrt{100 - 80}}{-8}$$

$$x = \frac{-10 \pm \sqrt{20}}{-8}$$

$$x = \frac{-10 \pm \sqrt{20}}{-8}$$

$$x = \frac{-10 \pm \sqrt{4.5}}{-8}$$

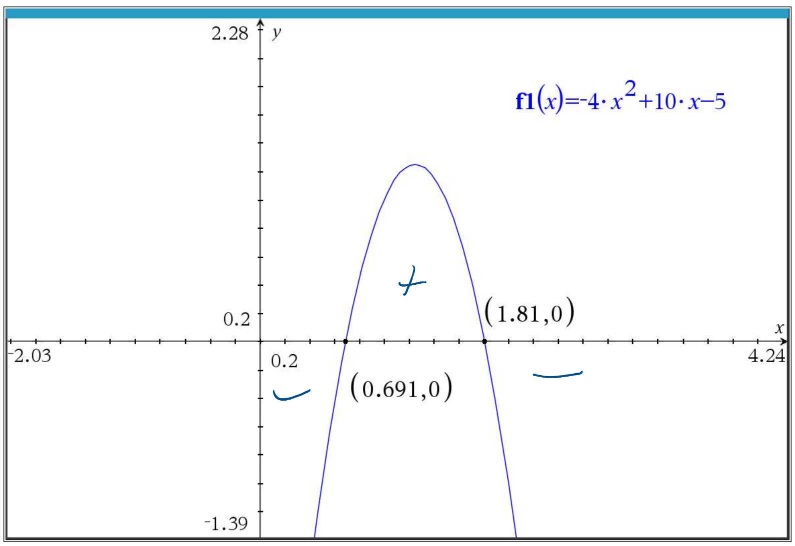
$$x = \frac{-10 \pm 2\sqrt{5}}{-8}$$

$$x = \frac{-5 \pm \sqrt{5}}{-4}$$

(-5-sqrt(5))/(-4)=1.809016994374947

(-5+sqrt(5))/(-4)=0.690983005625052

solution $\boxed{\left(\frac{-5-\sqrt{5}}{-4},\ \frac{-5+\sqrt{5}}{4}\right)}$



graph $\approx (0.691, 1.81)$

Algebraic method

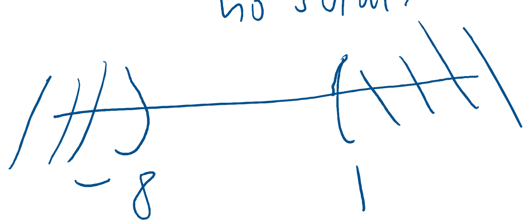
solve $f(x) < 0$

$$f(x) = (x-1)(x+8)$$

case 1 $\quad (x-1) > 0$ and $(x+8) < 0$

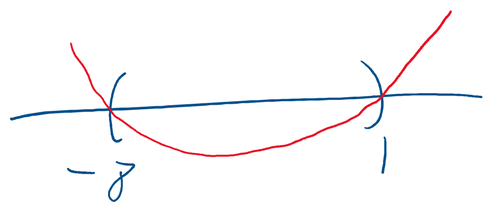
$x > 1$ and $x < -8$

no solution



case 2 $\quad x-1<0$ and $x+8>0$

$x<1$ and $x>-8$



solution $(-8, 1)$