2 Linear and Quadratic Functions

2.1 Linear Functions
2.1.1 Exercises
page 163: 5, 7, 15, 18, 26, 28, 32, 35, 45

2.2 Absolute Value Functions
2.2.1 Exercises
page 183: 1, 2, 15, 17, 22, 29

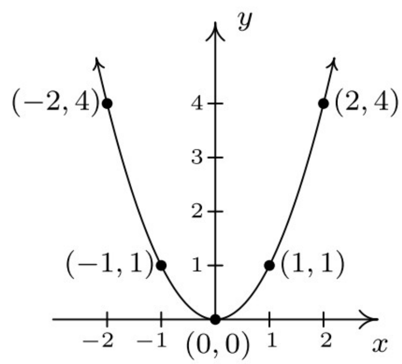
2.3 Quadratic Functions
2.3.1 Exercises
page 200: 1, 4, 11, 21, 26

2.3
Memorize

Definition 2.5. A **quadratic function** is a function of the form

$$f(x) = ax^2 + bx + c,$$

where a, b and c are real numbers with $a \neq 0$. The domain of a quadratic function is $(-\infty, \infty)$.



$f(x) = x^2$

The vertex here is (0,0). It is a local (relative) minimum for an upward opening parabola.
It is also the global minimum.

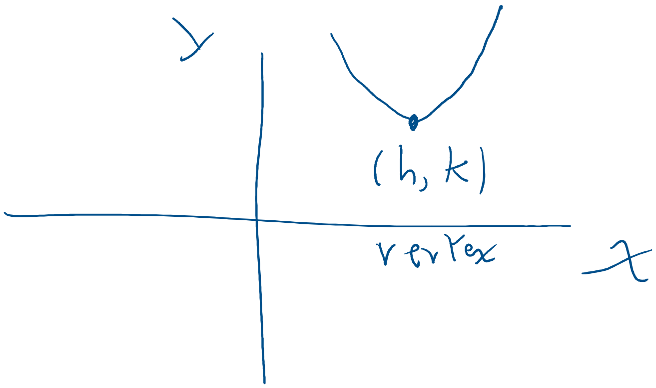
Range = $[0, \infty)$

Memorize

Definition 2.6. Standard and General Form of Quadratic Functions: Suppose f is a quadratic function.

- The **general form** of the quadratic function f is $f(x) = ax^2 + bx + c$, where a, b and c are real numbers with $a \neq 0$.
- vertex form
  The **standard form** of the quadratic function f is $f(x) = a(x-h)^2 + k$, where a, h and k are real numbers with $a \neq 0$.

$a > 0$



$a > 0$

$$f(h) = a(h-h)^2 + k$$
$$= a(0^2) + k$$
$$= 0 + k$$
$$= k$$

Convert the general form $f(x) = 2x^2 - 6x + 3$ to the vertex form by completing the square.

1) Group the x terms together

$$f(x) = (2x^2 - 6x) + 3$$

2) Factor out the coefficient of the x^2 term.

$$f(x) = 2(x^2 - 3x) + 3$$

3) Take 1/2 the coefficient of x, square it, and add and subtract the result inside the parentheses.

$$\left(\frac{1}{2}\right)(-3) = -\frac{3}{2}$$
$$\left(-\frac{3}{2}\right)^2 = \frac{9}{4}$$
$$f(x) = 2\left(x^2 - 3x + \frac{9}{4} - \frac{9}{4}\right) + 3$$

distributive property of multiplication over addition
$a(b+c) = ab + ac$

4) Remove the negative constant from inside the parentheses

$$f(x) = 2\left(x^2 - 3x + \frac{9}{4}\right) - (2)\left(\frac{9}{4}\right) + 3$$

5) Factor the perfect square inside the parentheses.

$$f(x) = 2\left(x - \frac{3}{2}\right)^2 - \frac{9}{2} + 3$$

$f(x) = 2(x - \frac{3}{2})^2 - \frac{3}{2}$

$$\boxed{f(x) = 2\left(x - \frac{3}{2}\right)^2 - \frac{3}{2}}$$ vertex form

vertex $= (h, k)$

$= \left(\frac{3}{2}, -\frac{3}{2}\right)$

$$(p+q)^2 = p^2 + 2pq + q^2$$

$$\begin{array}{r} (p+q)^2 = \quad p + q \\ \times \quad p + q \\ \hline pq + q^2 \\ p^2 + pq \quad \\ \hline p^2 + 2pq + q^2 \end{array}$$

$x^2 - 3x + ?$

$x = p \Rightarrow 3x = 3p$

$2pq = 3p$

$2qp = 3p$

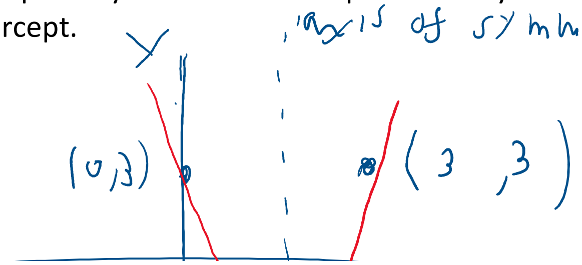
$2q = 3$

$q = \frac{3}{2}$

$q^2 = \frac{9}{4}$

Graph a parabola by completing the square (if necessary), finding and plotting the vertex, finding and plotting the y-intercept, finding and plotting a third point symmetric with respect to the y-intercept.

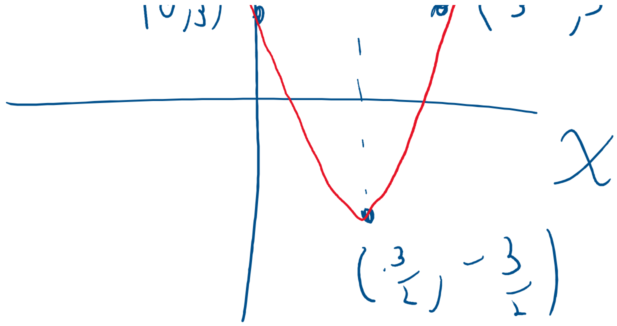
axis of symmetry



y-intercept

$f(0) = 2(0^2) - 6(0) + 3$

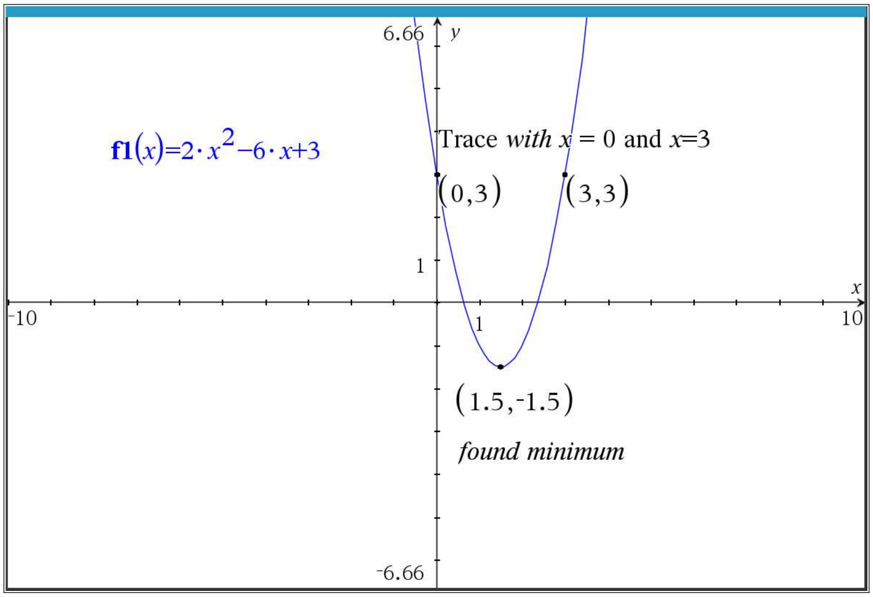
$= 3$



$f(0) = 2(0)^2 - 6(0) + 3$

$= 3$

3rd pt $\frac{3}{2} + \frac{3}{2} = \frac{6}{2} = 3$



Equation 2.4. Vertex Formulas for Quadratic Functions: Suppose a, b, c, h and k are real numbers with $a \neq 0$.

- If $f(x) = a(x-h)^2 + k$, the vertex of the graph of $y = f(x)$ is the point (h, k).
- If $f(x) = ax^2 + bx + c$, the vertex of the graph of $y = f(x)$ is the point $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

Memorize

Equation 2.5. The Quadratic Formula: If a, b and c are real numbers with $a \neq 0$, then the solutions to $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Memorize

Definition 2.7. If a, b and c are real numbers with $a \neq 0$, then the **discriminant** of the quadratic equation $ax^2 + bx + c = 0$ is the quantity $b^2 - 4ac$.

Theorem 2.3. Discriminant Trichotomy: Let a, b and c be real numbers with $a \neq 0$.

- If $b^2 - 4ac < 0$, the equation $ax^2 + bx + c = 0$ has no real solutions.
- If $b^2 - 4ac = 0$, the equation $ax^2 + bx + c = 0$ has exactly one real solution.
- If $b^2 - 4ac > 0$, the equation $ax^2 + bx + c = 0$ has exactly two real solutions.

2.3

Quiz 4 Your Name MTH 161-004N

4. $f(x) = -2(x+1)^2 + 4$

Find and plot the vertex, the y-intercept, and a 3rd point by symmetry. Connect the points to graph the parabola.

$f(x) = a(x-h)^2 + k$

vertex $= (h, k)$

$f(x) = -2(x-(-1))^2 + 4$

$h = -1$

$k = 4$

vertex $= (h, k) = (-1, 4)$

y-intercept $= f(0) = -2(0+1)^2 + 4 = -2(1^2) + 4 = (-2)(1) + 4$

$= -2 + 4 = 2$