

1.6 Graphs of Functions

1.6.2 Exercises

page 107: 1, 7, 9, 14, 21, 24, 32, 75

Exam 1		stem & leaf	
36.8	mean	9 6	A-1
34	median	8	B-0
21.10039	st. dev	7	C-0
15	min	6 9	D-1
96	max	5	F-13
15	count	4 078	
		3 447	
		2 0349	
		1 579	

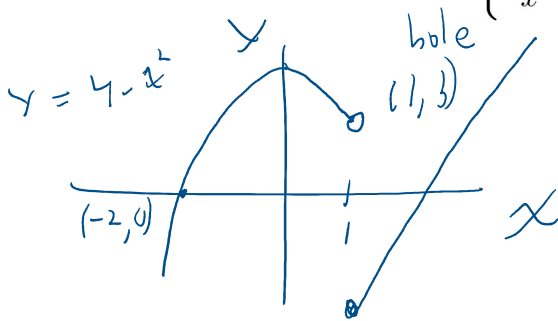
1.6

Memorize

The Fundamental Graphing Principle for Functions

The graph of a function f is the set of points which satisfy the equation $y = f(x)$. That is, the point (x, y) is on the graph of f if and only if $y = f(x)$.

Example 1.6.2. Graph: $f(x) = \begin{cases} 4 - x^2 & \text{if } x < 1 \\ x - 3, & \text{if } x \geq 1 \end{cases}$



$$y_1 = (4 - x^2)(x < 1) + (x - 3)(x \geq 1)$$

$$x < 1 \quad \underline{(4 - x^2)}(1) + (x - 3)(0)$$

$$x \geq 1 \quad (4 - x^2)(0) + (x - 3)(1)$$

$$x \geq 1 \quad (4 - x^2)(0) + (x - 3)(1)$$

Memorize

Definition 1.9. The **zeros** of a function f are the solutions to the equation $f(x) = 0$. In other words, x is a zero of f if and only if $(x, 0)$ is an x -intercept of the graph of $y = f(x)$.

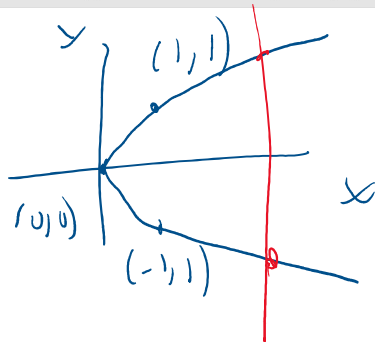
Memorize

Testing the Graph of a Function for Symmetry

The graph of a function f is symmetric

even
odd

- about the y -axis if and only if $f(-x) = f(x)$ for all x in the domain of f .
- about the origin if and only if $f(-x) = -f(x)$ for all x in the domain of f .



$$x = y^2$$

We cannot have x -axis symmetry for y as a function of x . Such a relation would fail the vertical line test

Google Gemini

Please give me three examples of testing a graph for symmetry.

◆ Absolutely! Here are three examples of testing a graph for symmetry, focusing on different types of symmetry:

1. Testing for Symmetry with Respect to the Y-Axis (Even Function)

- **Concept:** A graph is symmetric with respect to the y -axis if replacing x with $-x$ results in the same equation.
- **Example:** Consider the equation $y = x^2 + 2$.
 - **Test:** Replace x with $-x$:
 - $y = (-x)^2 + 2$
 - $y = x^2 + 2$
 - **Result:** The equation remains unchanged. Therefore, the graph of $y = x^2 + 2$ is symmetric with respect to the y -axis.

2. Testing for Symmetry with Respect to the X-Axis

- **Concept:** A graph is symmetric with respect to the x -axis if replacing y with $-y$ results in the same equation.
- **Example:** Consider the equation $x = y^2$.
 - **Test:** Replace y with $-y$:

2. Testing for Symmetry with Respect to the X-Axis

- **Concept:** A graph is symmetric with respect to the x -axis if replacing y with $-y$ results in the same equation.
- **Example:** Consider the equation $x = y^2$.
 - **Test:** Replace y with $-y$:
 - $x = (-y)^2$
 - $x = y^2$
 - **Result:** The equation remains unchanged. Therefore, the graph of $x = y^2$ is symmetric with respect to the x -axis.

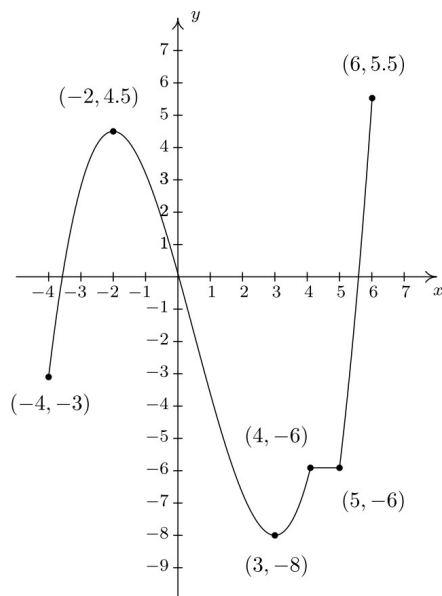
3. Testing for Symmetry with Respect to the Origin (Odd Function)

- **Concept:** A graph is symmetric with respect to the origin if replacing both x with $-x$ and y with $-y$ results in the same equation (or an equivalent equation where the entire equation has changed signs).
- **Example:** Consider the equation $y = x^3$.
 - **Test:** Replace x with $-x$ and y with $-y$:
 - $-y = (-x)^3$
 - $-y = -x^3$
 - $y = x^3$ (multiply both sides by -1)
 - **Result:** The equation, after manipulation, is the same as the original. Therefore, the graph of $y = x^3$ is symmetric with respect to the origin.

Memorize

Definition 1.10. Suppose f is a function defined on an interval I . We say f is:

- **increasing** on I if and only if $f(a) < f(b)$ for all real numbers a, b in I with $a < b$.
- **decreasing** on I if and only if $f(a) > f(b)$ for all real numbers a, b in I with $a < b$.
- **constant** on I if and only if $f(a) = f(b)$ for all real numbers a, b in I .



The graph of $y = f(x)$

Increasing on interval $(-4, -2)$
 $(3, 4)$
 $(5, 6)$
 Decreasing on interval $(-2, 3)$
 Constant on interval $(4, 5)$

Memorize

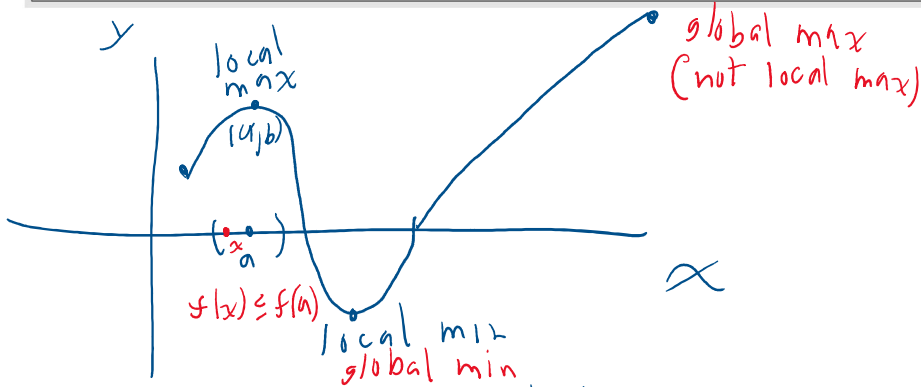
Definition 1.11. Suppose f is a function with $f(a) = b$.

relative

Memorize

Definition 1.11. Suppose f is a function with $f(a) = b$.

- We say f has a **local maximum** at the point (a, b) if and only if there is an open interval I containing a for which $f(a) \geq f(x)$ for all x in I . The value $f(a) = b$ is called 'a local maximum value of f ' in this case.
- We say f has a **local minimum** at the point (a, b) if and only if there is an open interval I containing a for which $f(a) \leq f(x)$ for all x in I . The value $f(a) = b$ is called 'a local minimum value of f ' in this case.
- The value b is called the **maximum** of f if $b \geq f(x)$ for all x in the domain of f .
- The value b is called the **minimum** of f if $b \leq f(x)$ for all x in the domain of f .



local max = top of a small hill
 local min = bottom of a small valley

