

Exam 1

Thursday 02/13/25

1.1 - 1.5

Graphing calculator required.

No scratch paper. No scantron. No bluebook.

Quiz 2 - take home - to be posted in Canvas Assignments.

Due Sunday, 02/09/25, 11:59 pm.

Open book, open notes.

Upload completed quiz in Canvas Assignments

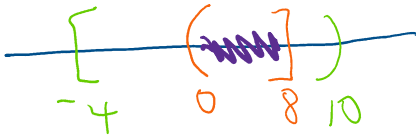
MTH 161-004N Quiz 2

Open book, open notes, graphing calculator allowed. Put a box around each answer.

1. Convert $\{x \mid 5 \leq x < 8\}$ into interval notation.

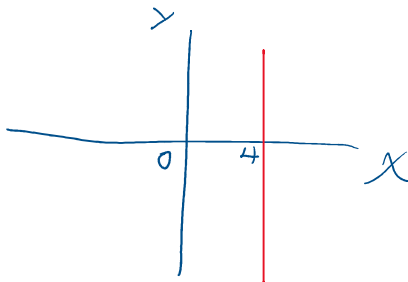
[5,8)

2. Simplify and write in interval notation: $[-4,10) \cap (0,8]$.



(0,8]

3. Graph the line given by the equation $x = 4$.



4. Let $R = \{(x,y) \mid x \geq 0 \text{ and } y \leq 10\}$. Does R represent y as a function of x ?
Why or why not?

No. $(1,3)$ and $(1,5)$ are two elements of the relation, with the same input but different outputs, violating the definition of y as a function of x .

$f(x) = \dots$ $f(1) = \dots$

outputs, violating the definition of y as a function of x .

5. Let $f(x) = 2 - x^2$. Find and simplify $f(2 + h)$.

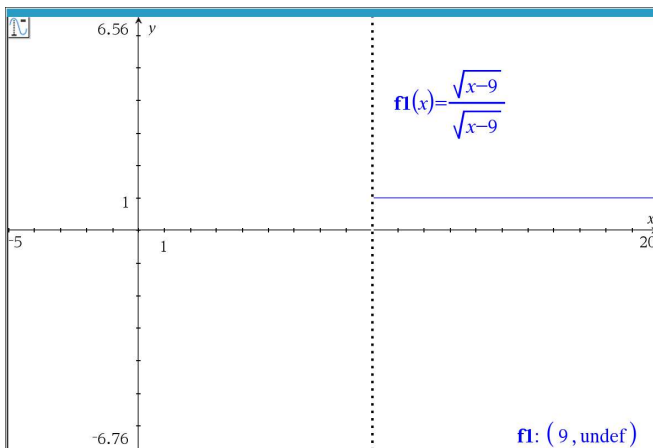
$$f(2 + h) = 2 - (2 + h)^2 = 2 - 4 - 4h - h^2 = \boxed{-2 - 4h - h^2}$$

6. Let $f(x) = g(x) = \sqrt{x - 9}$. Find the domain of $\left(\frac{f}{g}\right)(x)$

and then simplify $\left(\frac{f}{g}\right)(x)$.

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x-9}}{\sqrt{x-9}} = \boxed{1, \text{ for } x > 9}$$

$$\text{domain of } \left(\frac{f}{g}\right)(x) = \{x | x > 9\} = (9, \infty)$$



7. Find and simplify the difference quotient $\frac{\Delta f}{\Delta x}$ for $f(x) = x^2 + 2x$.

$$\begin{aligned} \frac{\Delta f}{\Delta x} &= \frac{f(x+h) - f(x)}{h} = \frac{[(x+h)^2 + 2(x+h)] - [x^2 + 2x]}{h} \\ &= \frac{[x^2 + 2hx + h^2 + 2x + 2h] - [x^2 + 2x]}{h} \\ &= \frac{x^2 + 2hx + h^2 + 2x + 2h - x^2 - 2x}{h} \\ &= \frac{2hx + h^2 + 2h}{h} \\ &= \frac{h(2x + h + 2)}{h} \end{aligned}$$

$$\boxed{\frac{\Delta f}{\Delta x} = 2x + h + 2}$$

8. What is the geometric meaning of the difference quotient?
Answer in one sentence.

The difference quotient of a function $f(x)$ is the slope of the secant line joining two points of the graph of $y = f(x)$.

Please give me a problem with calculating and simplifying the difference quotient of a function.

Here's a problem involving the difference quotient:

Problem:

Find and simplify the difference quotient for the function $f(x) = 2x^2 - 3x + 1$. The difference quotient is given by the formula:

$$\frac{f(x+h) - f(x)}{h}, \text{ where } h \neq 0.$$

Solution:

1. Find $f(x+h)$:

Substitute $(x+h)$ for x in the original function:

$$f(x+h) = 2(x+h)^2 - 3(x+h) + 1$$

$$f(x+h) = 2(x^2 + 2xh + h^2) - 3x - 3h + 1$$

$$f(x+h) = 2x^2 + 4xh + 2h^2 - 3x - 3h + 1$$

2. Find $f(x+h) - f(x)$:

Subtract the original function $f(x)$ from $f(x+h)$:

$$f(x+h) - f(x) = (2x^2 + 4xh + 2h^2 - 3x - 3h + 1) - (2x^2 - 3x + 1)$$

$$f(x+h) - f(x) = 2x^2 + 4xh + 2h^2 - 3x - 3h + 1 - 2x^2 + 3x - 1$$

$$f(x+h) - f(x) = 4xh + 2h^2 - 3h$$

3. Divide by h :

Divide the result from step 2 by h :

$$\frac{f(x+h) - f(x)}{h} = \frac{4xh + 2h^2 - 3h}{h}$$

4. Simplify:

Factor out an h from the terms in the numerator and then cancel it with the h in the denominator:

$$\frac{f(x+h) - f(x)}{h} = \frac{h(4x + 2h - 3)}{h}$$

$$\frac{f(x+h) - f(x)}{h} = 4x + 2h - 3$$

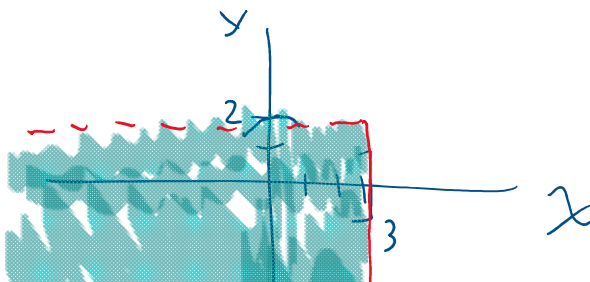
Therefore, the simplified difference quotient for $f(x) = 2x^2 - 3x + 1$ is $4x + 2h - 3$.

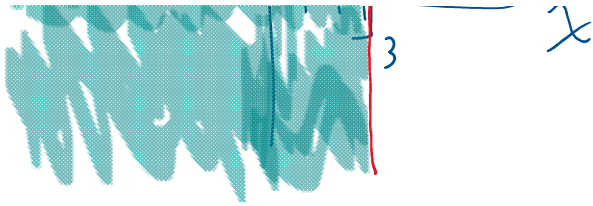
Co-pilot confused this type of problem with a related calculus problem

1.2.2 EXERCISES

In Exercises 1 - 20, graph the given relation.

18. $\{(x, y) \mid x \leq 3, y < 2\}$





1.2: 50

For each equation given in Exercises 41 - 52:

- Find the x - and y -intercept(s) of the graph, if any exist.
- Follow the procedure in Example 1.2.3 to create a table of sample points on the graph of the equation.
- Plot the sample points and create a rough sketch of the graph of the equation.
- Test for symmetry. If the equation appears to fail any of the symmetry tests, find a point on the graph of the equation whose reflection fails to be on the graph as was done at the end of Example 1.2.4

50. $x^2 - y^2 = 1$

x -intercept
set $y=0$, solve for x

$$x^2 - 0^2 = 1$$

$$x^2 = 1$$

$$x = \pm 1$$

or the points
 $(-1, 0), (1, 0)$

y -intercept
set $x=0$, solve for y

$$0^2 - y^2 = 1$$

$$-y^2 = 1$$

$$y^2 = -1$$

$$y = \pm \sqrt{-1} = \pm i$$

non-real

\therefore No y -intercept

$$x^2 - y^2 = 1$$

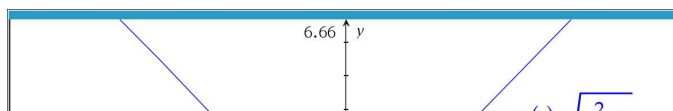
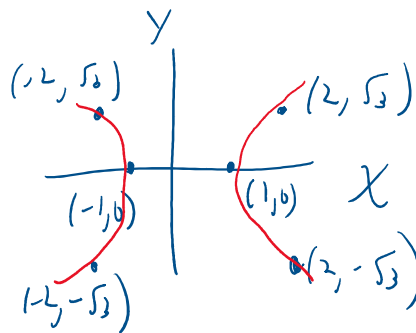
solve for y

$$-y^2 = 1 - x^2$$

$$y^2 = x^2 - 1$$

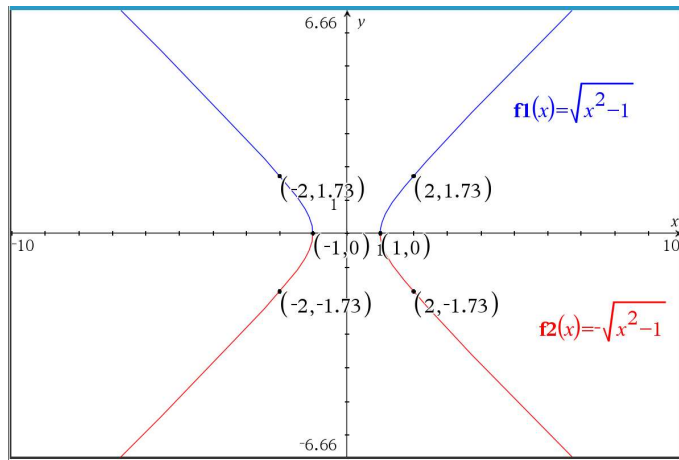
$$y = \pm \sqrt{x^2 - 1}$$

x	y
-1	0
1	0
2	$-\sqrt{3}$
2	$\sqrt{3}$
-2	$-\sqrt{3}$
-2	$\sqrt{3}$



$$-2 \sqrt{5}$$

Sqrt(3)=
1.732050807568877



Test for symmetry

$$\begin{array}{l} \text{same} \left(\begin{array}{l} x^2 - y^2 = 1 \\ x^2 - (-y)^2 = 1 \\ x^2 - y^2 = 1 \end{array} \right) \\ \therefore x\text{-axis symmetry} \end{array}$$

$$\left. \begin{array}{l} x^2 - y^2 = 1 \\ (-x)^2 - y^2 = 1 \\ x^2 - y^2 = 1 \end{array} \right\} \text{same} \\ y\text{-axis symmetry}$$

$$\left. \begin{array}{l} x^2 - y^2 = 1 \\ (-x)^2 - (-y)^2 = 1 \\ x^2 - y^2 = 1 \end{array} \right\} \text{same} \\ \therefore \text{origin symmetry}$$

1.5:1

1.5.1 EXERCISES

In Exercises 1 - 10, use the pair of functions f and g to find the following values if they exist.

- $(f + g)(2)$

- $(f - g)(-1)$

- $(g - f)(1)$

- $(fg)\left(\frac{1}{2}\right)$

- $\left(\frac{f}{g}\right)(0)$

- $\left(\frac{g}{f}\right)(-2)$

1. $f(x) = 3x + 1$ and $g(x) = 4 - x$

$$\begin{aligned} (fg)\left(\frac{1}{2}\right) &= f\left(\frac{1}{2}\right) \cdot g\left(\frac{1}{2}\right) \\ &= \left[3\left(\frac{1}{2}\right) + 1\right] \left[4 - \frac{1}{2}\right] \\ &= \left(\frac{5}{2}\right) \left(\frac{7}{2}\right) \\ &= \boxed{\frac{35}{4}} \end{aligned}$$

$$(fg)(x) = (3x+1)(4-x)$$

$$\begin{aligned}
 (fg)(x) &= (3x+1)(4-x) \\
 &= 12x - 3x^2 + 4 - x \\
 (fg)(x) &= -3x^2 + 11x + 4 \\
 fg\left(\frac{1}{2}\right) &= -3\left(\frac{1}{2}\right)^2 + 11\left(\frac{1}{2}\right) + 4 \\
 &= -\frac{3}{4} + \frac{22}{4} + \frac{16}{4} \\
 &= \boxed{\frac{35}{4}}
 \end{aligned}$$

1.5: 46

In Exercises 46 - 50, $C(x)$ denotes the cost to produce x items and $p(x)$ denotes the price-demand function in the given economic scenario. In each Exercise, do the following:

- Find and interpret $C(0)$.
- Find and interpret $\bar{C}(10)$.
- Find and interpret $p(5)$.
- Find and simplify $R(x)$.
- Find and simplify $P(x)$.
- Solve $P(x) = 0$ and interpret.

46. The cost, in dollars, to produce x "I'd rather be a Sasquatch" T-Shirts is $C(x) = 2x + 26$, $x > 0$ and the price-demand function, in dollars per shirt, is $p(x) = 30 - 2x$, $0 < x < 15$.

Summary of Common Economic Functions

Suppose x represents the quantity of items produced and sold.

- The price-demand function $p(x)$ calculates the price per item.
- The revenue function $R(x)$ calculates the total money collected by selling x items at a price $p(x)$, $R(x) = xp(x)$.
- The cost function $C(x)$ calculates the cost to produce x items. The value $C(0)$ is called the fixed cost or start-up cost.
- The average cost function $\bar{C}(x) = \frac{C(x)}{x}$ calculates the cost per item when making x items. Here, we necessarily assume $x > 0$.
- The profit function $P(x)$ calculates the money earned after costs are paid when x items are produced and sold, $P(x) = (R - C)(x) = R(x) - C(x)$.

Read the corresponding example 1.5.3