02-06-25 MTH 161

1.5 Function Arithmetic

1.5.1 Exercises

page 84: 1, 11, 17, 21,23,25, 46, 57

Exam 1

Thursday 02/13/25

1.1 - 1.5

Graphing calculator required.

No scratch paper. No scantron. No bluebook.

Quiz 2 - take home - to be posted in Canvas Assignments.

Due Sunday, 02/09/25, 11:59 pm.

Open book, open notes.

Upload completed quiz in Canvas Assignments

1.5: 17

In Exercises 11 - 20, use the pair of functions f and g to find the domain of the indicated function then find and simplify an expression for it.

- $(f+g)(x)$
- $(f-g)(x)$
- $(fg)(x)$
- $\left(\frac{f}{g}\right)(x)$

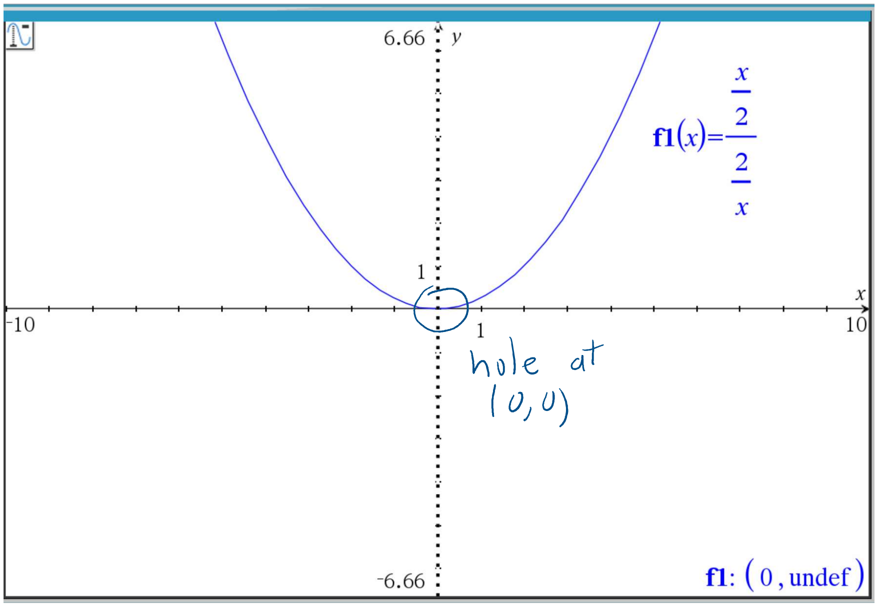
17. $f(x) = \frac{x}{2}$ and $g(x) = \frac{2}{x}$

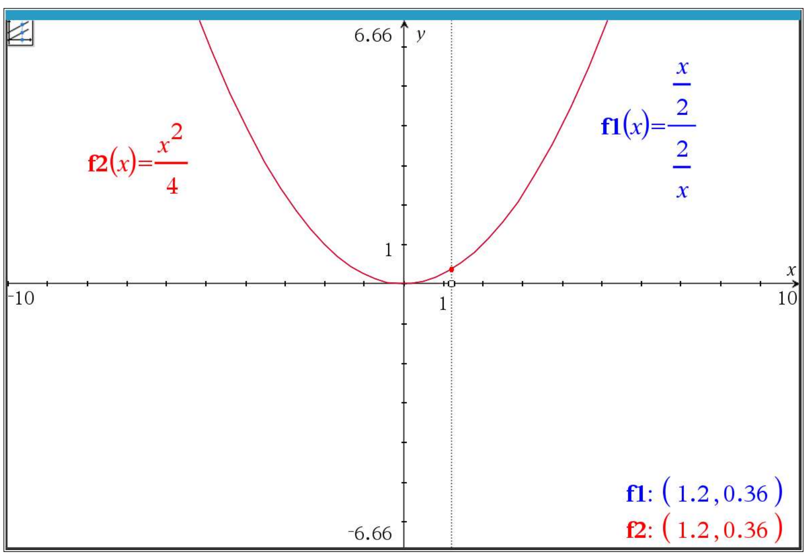
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \boxed{\frac{\frac{x}{2}}{\frac{2}{x}}} = \frac{x}{2} \cdot \frac{x}{2} = \boxed{\frac{x^2}{4}}$$

invert denominator and multiply

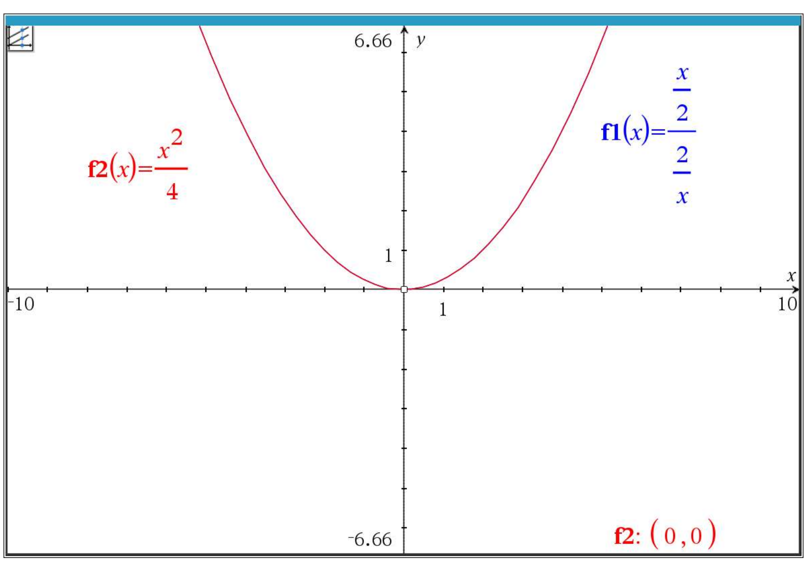
To find the domain, consider the unsimplified form

$$\text{domain} = \{x \mid x \neq 0\}$$
$$= (-\infty, 0) \cup (0, \infty)$$





For $x \neq 0$, the two functions are equal



When tracing both functions, the unsimplified (blue) function disappears when x = 0.

1.5: 46

In Exercises 46 - 50, $C(x)$ denotes the cost to produce x items and $p(x)$ denotes the price-demand function in the given economic scenario. In each Exercise, do the following:

- Find and interpret $C(0)$.
- Find and interpret $\overline{C}(10)$.
- Find and interpret $p(5)$
- Find and simplify $R(x)$.
- Find and simplify $P(x)$.
- Solve $P(x) = 0$ and interpret.

46. The cost, in dollars, to produce x "I'd rather be a Sasquatch" T-Shirts is $C(x) = 2x + 26$, $x \geq 0$ and the price-demand function, in dollars per shirt, is $p(x) = 30 - 2x$, $0 \leq x \leq 15$.

Supplied

Summary of Common Economic Functions

Suppose x represents the quantity of items produced and sold.

- The price-demand function $p(x)$ calculates the price per item.
- The revenue function $R(x)$ calculates the total money collected by selling x items at a price $p(x)$, $R(x) = x\,p(x)$.
- The cost function $C(x)$ calculates the cost to produce x items. The value $C(0)$ is called the fixed cost or start-up cost.
- The average cost function $\overline{C}(x) = \frac{C(x)}{x}$ calculates the cost per item when making x items. Here, we necessarily assume $x > 0$.
- The profit function $P(x)$ calculates the money earned after costs are paid when x items are produced and sold, $P(x) = (R - C)(x) = R(x) - C(x)$.

$$P(x) = R(x) - C(x) = x\,p(x) - C(x)$$

$$C(0) = 2(0) + 26 = 0 + 26 = \boxed{26 = C(0)}$$

Interpretation: The cost of producing 0 T-shirts is \$26.

$$\overline{C}(10) = \frac{C(10)}{10} = \frac{2(10)+26}{10} = \frac{20+26}{10} = \frac{46}{10} = \boxed{4.6 = \overline{C}(10)}$$

Interpretation: The average cost of producing 10 T-shirts is \$4.60.

$$p(5) = 30 - 2(5) = 30 - 10 = \boxed{20}$$

Interpretation: To sell 5 shirts, set the price at \$20 per shirt.

$$R(x) = x\,p(x)$$
$$= x(30 - 2x)$$
$$\boxed{R(x) = 30x - 2x^2}$$

$= x(30 - 2x)$

$$\boxed{R(x) = 30x - 2x^2}$$

$$P(x) = x\,p(x) - C(x)$$

$$= x(30-2x) - (2x+26)$$

$$= 30x - 2x^2 - 2x - 26$$

$$\boxed{P(x) = -2x^2 + 28x - 26}$$

$$P(x) = 0$$

$$\Leftrightarrow \quad -2x^2 + 28x - 26 = 0$$

solve for x

$$\frac{-2x^2}{-2} + \frac{28x}{-2} - \frac{26}{-2} = \frac{0}{-2}$$

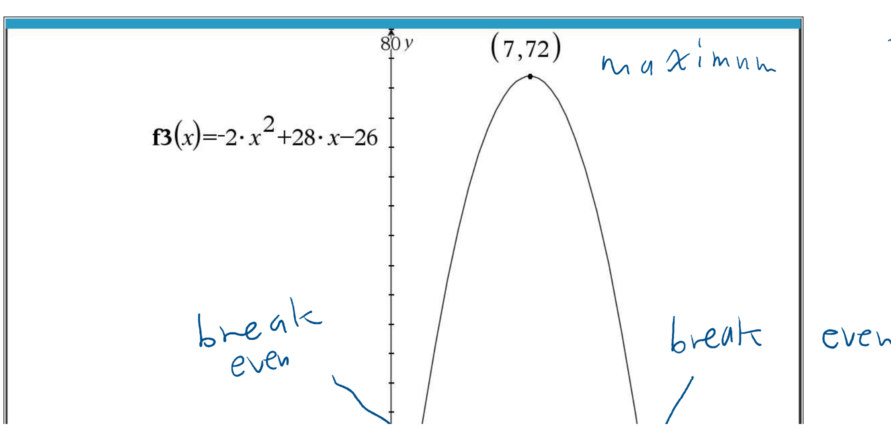
$$x^2 - 14x + 13 = 0$$

$$(x-13)(x-1) = 0$$

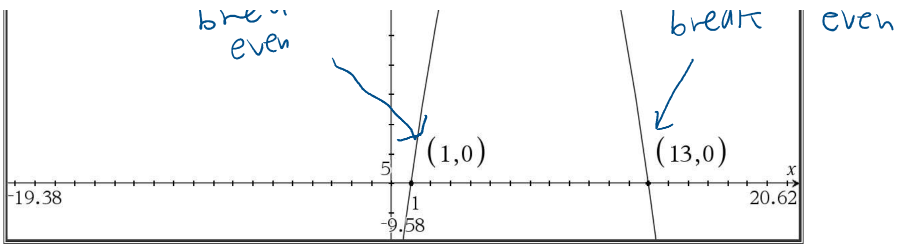
$$x - 13 = 0 \quad \text{or} \quad x - 1 = 0$$

$$\boxed{x = 13, 1}$$

Interpretation: If we sell 1 or 13 T-shirts, we break even.



max profit = $72 when 7 T-shirts produced and sold.



If TI said $(1, 2.316\,E^{-15})$ instead of $(1, 0)$

$$2.316 \times 10^{-15} = \frac{2.316}{1000000000000000} \approx 0$$

1.5: 57

In Exercises 51 - 62, let f be the function defined by

$$f = \{(-3,4), \underline{(-2,2)}, (-1,0), (0,1), (1,3), (2,4), (3,-1)\}$$

$f(-2) = 2$

and let g be the function defined

$$g = \{(-3,-2), \underline{(-2,0)}, (-1,-4), (0,0), (1,-3), (2,1), (3,2)\}$$

$g(-2) = 0$

. Compute the indicated value <u>if it exists.</u>

57. $\left(\frac{f}{g}\right)(-2)$

$$= \frac{f(-2)}{g(-2)} = \frac{2}{0} \text{ not defined!}$$

Thus, $\left(\frac{f}{g}\right)(-2)$ does not exist