02-04-25 MTH 161-004N

1.3 Introduction to Functions
1.3.1 Exercises
page 43: 1, 2, 6, 14, 16, 39, 46

1.4 Function Notation
1.4.2 Exercises
page 63: 1, 4, 15, 19, 31, 35, 37, 39, 67

1.5 Function Arithmetic
1.5.1 Exercises
page 84: 1, 11, 17, 21,23,25, 46, 57

1.4: 35

35. Let $f(x) = \begin{cases} x+5 & \text{if } x \le -3 \\ \sqrt{9-x^2} & \text{if } -3 < x \le 3 \\ -x+5 & \text{if } x > 3 \end{cases}$ Compute the following function values.

(a) $f(-4)$ (b) $f(-3)$ (c) $f(3)$

(d) $f(3.001)$ (e) $f(-3.001)$ (f) $f(2)$

(a) $-4 \le -3 \Rightarrow$ use 1st formula

$f(-4) = -4+5 = \boxed{1}$

(b) $-3 \le -3 \Rightarrow$ use 1st formula

$f(-3) = -3+5 = \boxed{2}$

(c) $-3 < 3 \le 3 \Rightarrow$ use 2nd formula

$f(3) = \sqrt{9-3^2} = \sqrt{9-9} = \sqrt{0} = \boxed{0}$

(d) $3.001 > 3 \Rightarrow$ use 3rd formula

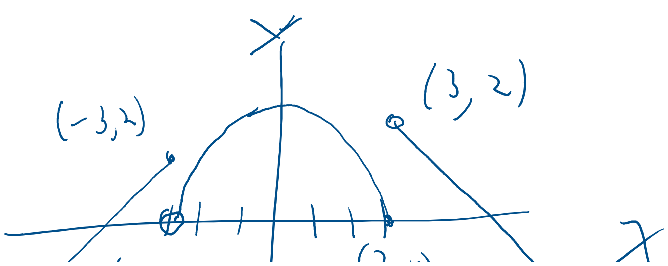
$f(3.001) = -3.001+5 = 1.999$

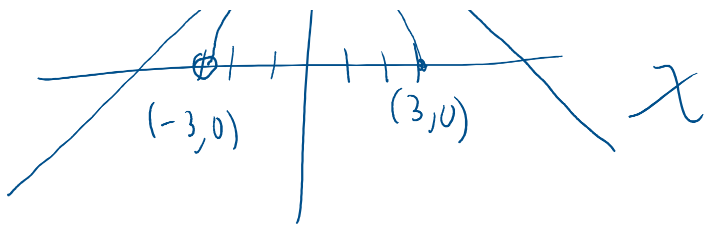
$y = \sqrt{9-x^2}$

$y^2 = 9-x^2$

$x^2+y^2=9$

circle
center $(0,0)$
radius $=3$

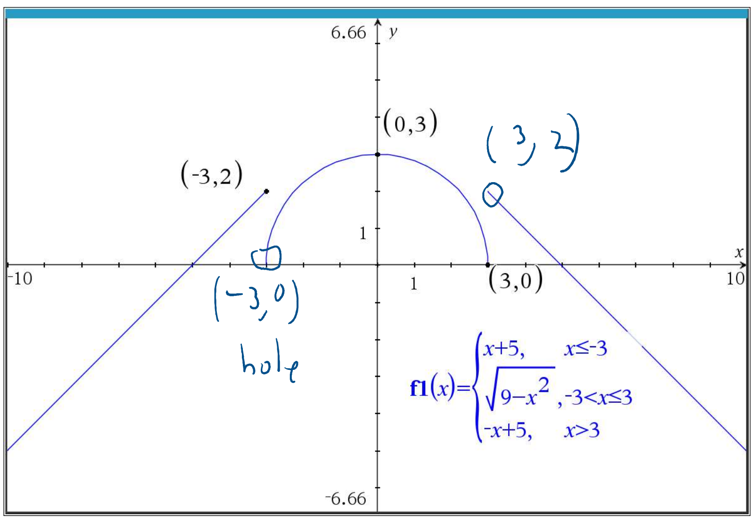




$$Y_1 = (x+5)(x \le -3) + \sqrt{9-x^2}\,(x > -3)$$

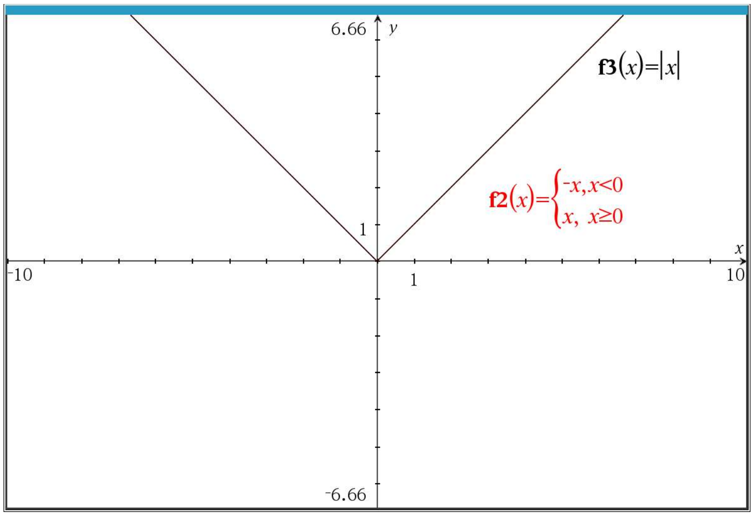
$$x \le -3 \Rightarrow Y_1 = (x+5)(1) + \sqrt{9-x^2}\,(0)$$

$$= x+5$$



$$Y_1 = -x(x<0) + x(x \ge 0)$$

$$f(x) = \begin{cases} -x, & x<0 \\ x, & x \ge 0 \end{cases}$$



1.4:37

In Exercises 37 - 62, find the (implied) domain of the function.

The (implied) domain of a function is the largest possible set of real numbers for which the function is well-defined, i.e., for which the function can be evaluated.

37. $f(x) = x^4 - 13x^3 + 56x^2 - 19$

domain = $(-\infty, \infty) = \{x | x \in \mathbb{R}\}$

As long as we substitute numbers other than 3 and -3, the expression $r(x)$ is a real number. Hence, we write our domain in interval notation[1] as $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$. When a formula for a function is given, we assume that the function is valid for all real numbers which make arithmetic sense when substituted into the formula. This set of numbers is often called the **implied domain**[2] of the function. At this stage, there are only two mathematical sins we need to avoid: division by 0 and extracting even roots of negative numbers. The following example illustrates these concepts.

Let's assume that we can divide by 0

$\frac{1}{0} = x$ for some $x \in \mathbb{R}$

$\Rightarrow 1 = 0 \cdot x$

$\boxed{1 = 0}$

$\boxed{1 = 0}$

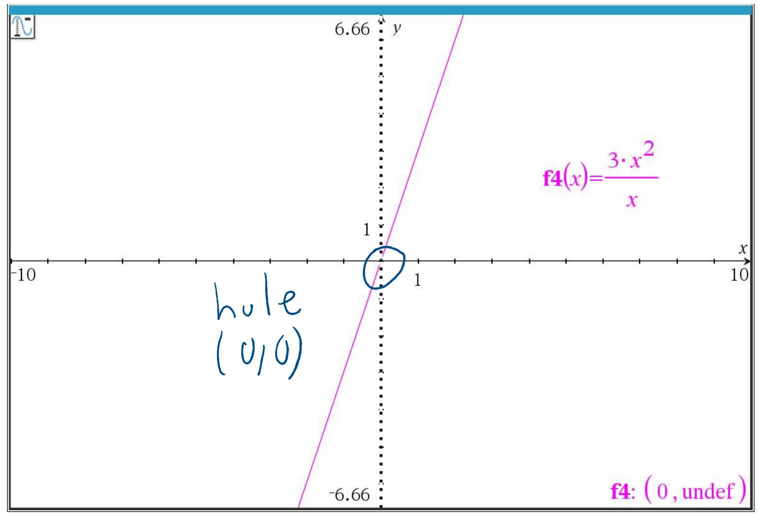
$1+1 = 0+1$

$2 = 1 = 0$

$2+1 = \boxed{3 = 0} + 1 = 1 = 0$

$\vdots$

$1{,}000{,}000 = 0$



domain of $f(x) = \frac{3x^2}{x} = \{x \mid x \neq 0\}$

$\neq$ domain of simplified $f(x) = 3x$
$= (-\infty, \infty)$

1.5
Memorize

Function Arithmetic

Suppose f and g are functions and x is in both the domain of f and the domain of g.[a]

- The **sum** of f and g, denoted $f+g$, is the function defined by the formula

$$(f+g)(x) = f(x) + g(x)$$

- The **difference** of f and g, denoted $f-g$, is the function defined by the formula

$$(f-g)(x) = f(x) - g(x)$$

- The **product** of f and g, denoted fg, is the function defined by the formula

$$(fg)(x) = f(x)g(x)$$

- The **quotient** of f and g, denoted $\frac{f}{g}$, is the function defined by the formula

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)},$$

provided $g(x) \neq 0$.

[a] Thus x is an element of the intersection of the two domains.

$f(x) = 3x$

$g(x) = \frac{1}{x}$

Let $h(x) = (f+g)(x)$

$= f(x) + g(x)$

$h(x) = 3x + \frac{1}{x}$

domain of $f+g$ = (domain f) $\cap$ (domain g)

Memorize

Definition 1.8. Given a function f, the **difference quotient** of f is the expression

$$\frac{f(x+h) - f(x)}{h}, \quad h \neq 0$$

Note: This is vital for calculus.

Let $f(x) = 2x+1$

Find and simplify the difference quotient $\frac{\Delta f}{\Delta x}$

Δ = change

Δ = change

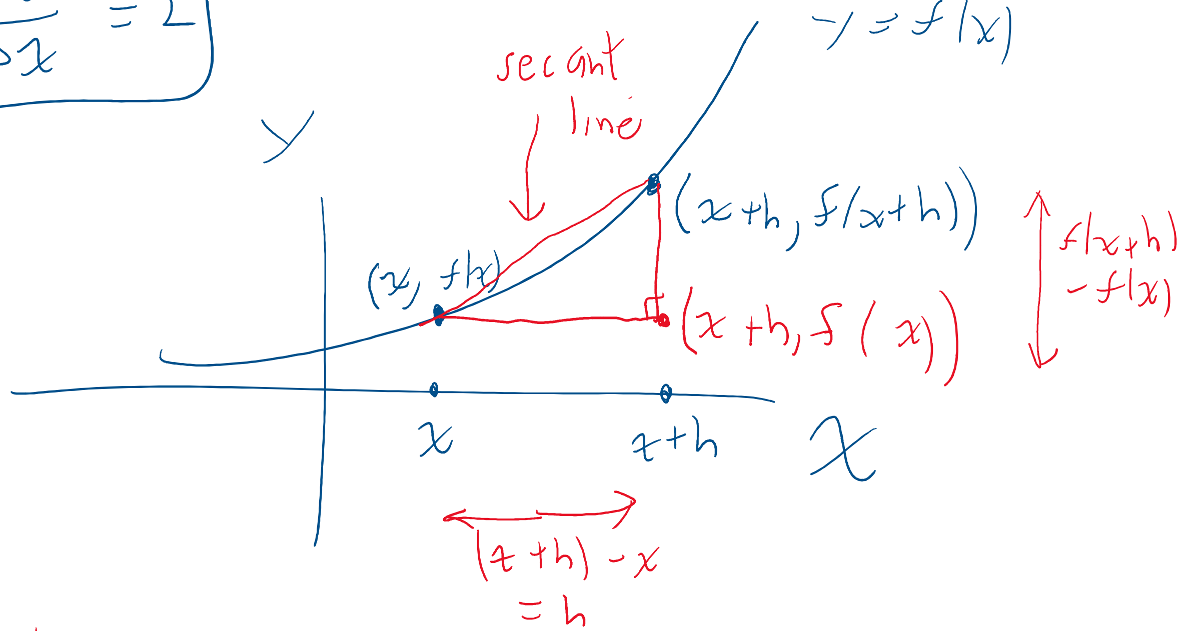
$$\frac{\Delta f}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{[2(x+h)+1] - (2x+1)}{h}$$

$$= \frac{\cancel{2x} + 2h \cancel{+1} - \cancel{2x} \cancel{-1}}{h}$$

$$= \frac{2\cancel{h}}{\cancel{h}}$$

$$\boxed{\frac{\Delta f}{\Delta x} = 2}$$



slope of secant line $= \frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{h}$

Supplied

Summary of Common Economic Functions

Suppose x represents the quantity of items produced and sold.

- The price-demand function $p(x)$ calculates the price per item.
- The revenue function $R(x)$ calculates the total money collected by selling x items at a price $p(x)$, $R(x) = x\,p(x)$.
- The cost function $C(x)$ calculates the cost to produce x items. The value $C(0)$ is called the fixed cost or start-up cost.
- The average cost function $\overline{C}(x) = \frac{C(x)}{x}$ calculates the cost per item when making x items. Here, we necessarily assume $x > 0$.
- The profit function $P(x)$ calculates the money earned after costs are paid when x items are produced and sold, $P(x) = (R - C)(x) = R(x) - C(x)$.