01-23-25 MTH 161

1 Relations and Functions
1.1 Sets of Real Numbers and the Cartesian Coordinate Plane
1.1.4 Exercises
page 14: 1, 3, 5, 11, 17, 23, 31

1.1
Memorize

Definition 1.1. A **set** is a well-defined collection of objects which are called the 'elements' of the set. Here, 'well-defined' means that it is possible to determine if something belongs to the collection or not, without prejudice.

Ways to Describe Sets

1. **The Verbal Method:** Use a sentence to define a set.
2. **The Roster Method:** Begin with a left brace '{', list each element of the set *only once* and then end with a right brace '}'. list
3. **The Set-Builder Method:** A combination of the verbal and roster methods using a "dummy variable" such as x.

n $A = \{2, 4, 6\} = \{6, 4, 2\}$

list notation

verbal = A = set of all even whole numbers greater than 1 and less than 7 $= \{6, 4, 2, 4\}$

$x \in A$ x is an element (member) of set A

$x \notin A$ x is not an element of set A

$2 \in A$, $5 \notin A$

set-builder $A = \{x \mid x \text{ is an even whole number and } 1 < x < 7\}$

$A = \{x \mid x \text{ is an even integer, and } 2 \le x \le 6\}$

B = set of all stars in our galaxy

$C = \{$coffee, Virginia, π, 3, dog$\}$

Memorize

Sets of Numbers

1. The **Empty Set**: $\emptyset = \{\} = \{x \mid x \neq x\}$. This is the set with no elements. Like the number '0,' it plays a vital role in mathematics.[a]

2. The **Natural Numbers**: $\mathbb{N} = \{1, 2, 3, \ldots\}$ The periods of ellipsis here indicate that the natural numbers contain 1, 2, 3, 'and so forth'.

3. The **Whole Numbers**: $\mathbb{W} = \{0, 1, 2, \ldots\}$

4. The **Integers**: $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

5. The **Rational Numbers**: $\mathbb{Q} = \left\{\frac{a}{b} \mid a \in \mathbb{Z} \text{ and } b \in \mathbb{Z}\right\}$. Rational numbers are the ratios of integers (provided the denominator is not zero!) It turns out that another way to describe the rational numbers[b] is:

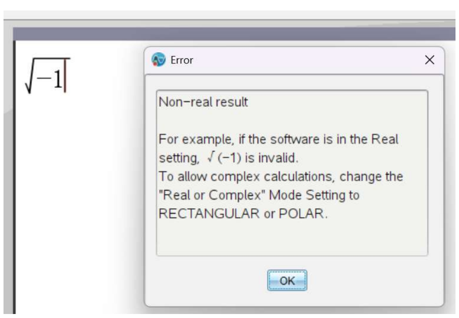
$$\mathbb{Q} = \{x \mid x \text{ possesses a repeating or terminating decimal representation.}\}$$

6. The **Real Numbers**: $\mathbb{R} = \{x \mid x \text{ possesses a decimal representation.}\}$

7. The **Irrational Numbers**: $\mathbb{P} = \{x \mid x \text{ is a non-rational real number.}\}$ Said another way, an irrational number is a decimal which neither repeats nor terminates.[c]

8. The **Complex Numbers**: $\mathbb{C} = \{a + bi \mid a, b \in \mathbb{R} \text{ and } i = \sqrt{-1}\}$ Despite their importance, the complex numbers play only a minor role in the text.[d]

[a] ... which, sadly, we will not explore in this text.
[b] See Section 9.2.
[c] The classic example is the number π (See Section 10.1), but numbers like $\sqrt{2}$ and 0.101001000100001... are other fine representatives.
[d] They first appear in Section 3.4 and return in Section 11.7.

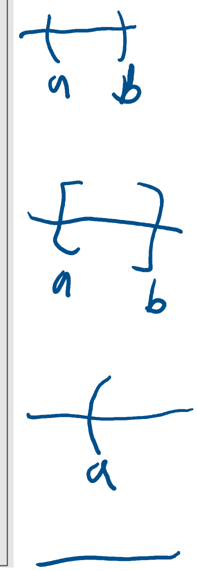


i^2 -1

Interval Notation

Let a and b be real numbers with $a < b$.

Set of Real Numbers	Interval Notation	Region on the Real Number Line
$\{x \mid a < x < b\}$	(a, b)	a b
$\{x \mid a \leq x < b\}$	$[a, b)$	a b
$\{x \mid a < x \leq b\}$	$(a, b]$	a b
$\{x \mid a \leq x \leq b\}$	$[a, b]$	a b
$\{x \mid x < b\}$	$(-\infty, b)$	b
$\{x \mid x \leq b\}$	$(-\infty, b]$	b
$\{x \mid x > a\}$	(a, ∞)	a
$\{x \mid x \geq a\}$	$[a, \infty)$	a
$\mathbb{R}$	$(-\infty, \infty)$	

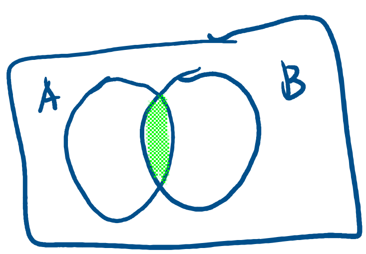


Memorize

Definition 1.2. Suppose A and B are two sets.

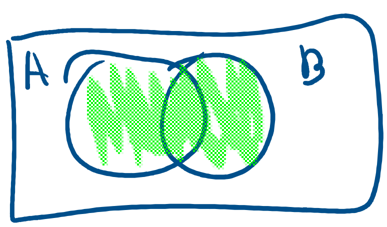
- The **intersection** of A and B: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
- The **union** of A and B: $A \cup B = \{x \mid x \in A \text{ or } x \in B \text{ (or both)}\}$

Venn diagram



U = set of discourse

$A \cap B$



U

Def. $A \subseteq B$

A is a subset of B

$(x \in A) \Rightarrow (x \in B)$

implies

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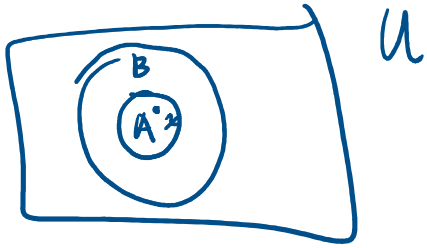
implies



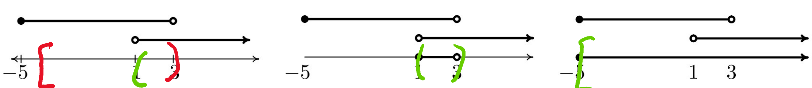
Def $A \subset B$

A is a <u>proper</u> subset of B

$A \subseteq B$ but $A \neq B$

Def $A = B$ if A and B have exactly the same elements

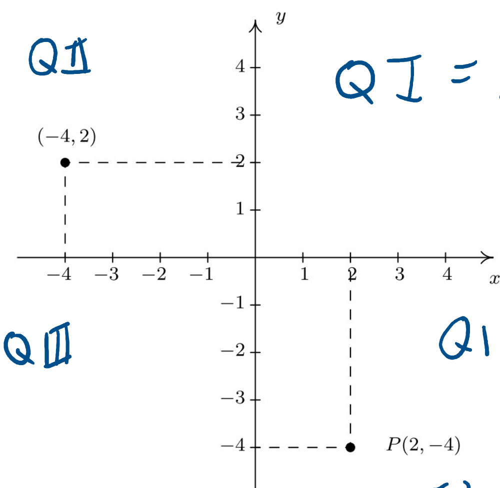
Theorem: $A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$



$A = [-5, 3)$, $B = (1, \infty)$ $\quad$ $A \cap B = (1, 3)$ $\quad$ $A \cup B = [-5, \infty)$

QII

QI = quadrant I

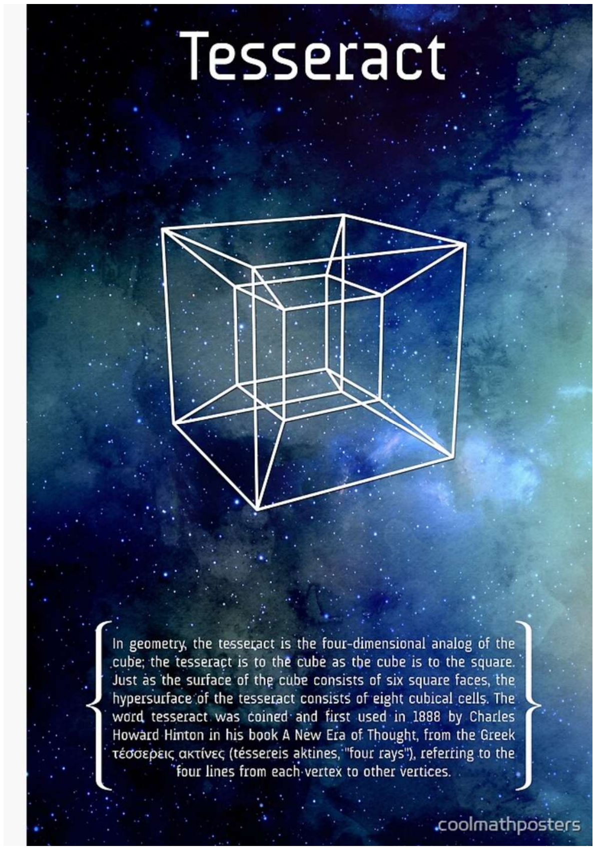


QIII

QIV

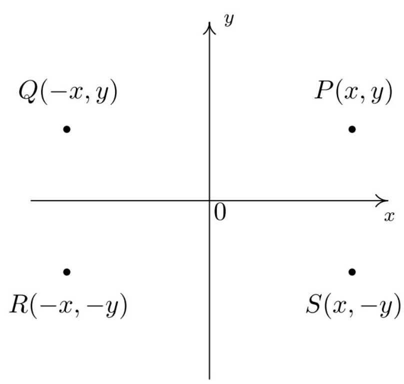
$x = 2$

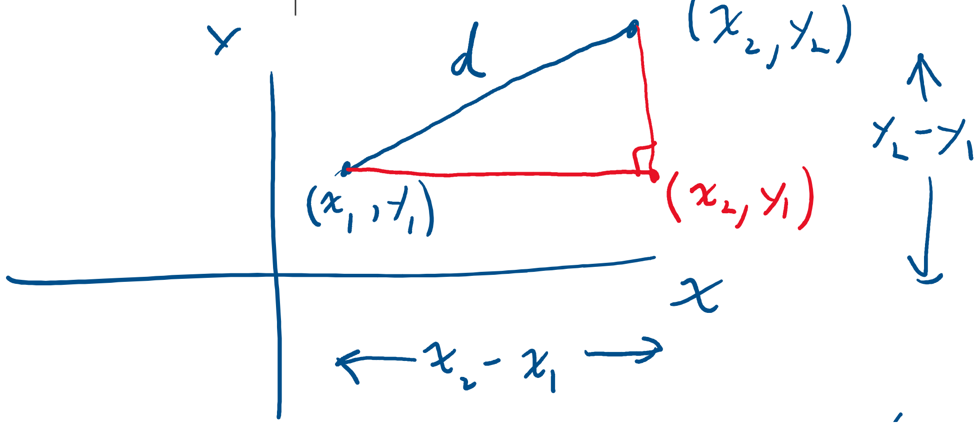
$y = -4$



Definition 1.3. Two points (a, b) and (c, d) in the plane are said to be

- **symmetric about the x-axis** if $a = c$ and $b = -d$
- **symmetric about the y-axis** if $a = -c$ and $b = d$
- **symmetric about the origin** if $a = -c$ and $b = -d$





$\leftarrow x_2 - x_1 \rightarrow$

$(x_2-x_1)^2+(y_2-y_1)^2=d^2$ [Pyth Thm.]

$$\boxed{d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}$$ memorize

distance formula

Memorize

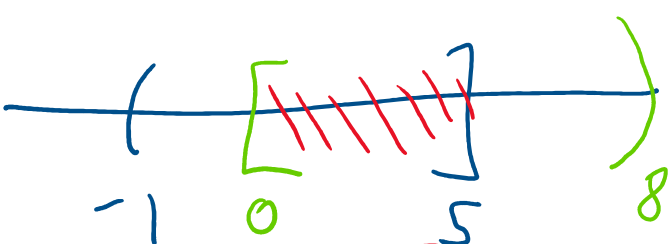
Equation 1.2. The Midpoint Formula: The midpoint M of the line segment connecting $P\,(x_0, y_0)$ and $Q\,(x_1, y_1)$ is:

$$M=\left(\frac{x_0+x_1}{2},\frac{y_0+y_1}{2}\right)$$

1.1

In Exercises 2 - 7, find the indicated intersection or union and simplify if possible. Express your answers in interval notation.

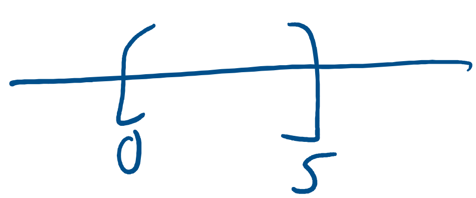
2. $(-1,5]\cap[0,8)$



$\boxed{[0,5]}$ interval notation

set-builder notation

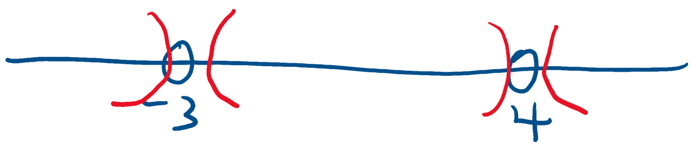
$\{x\in\mathbb{R} \mid 0\le x\le 5\}$



graph

In Exercises 8 - 19, write the set using interval notation.

10. $\{x \mid x \neq -3, 4\}$



graph

$$\boxed{(-\infty, -3) \cup (-3, 4) \cup (4, \infty)}$$