

Chapter 7 - Regular Polygons and Circles

7.1 Regular Polygons, p. 245: 1, 7, 9

7.2: Circles, p. 255: 1, 3

7.4 Degrees in an Arc, p. 277: 1. 5

7.5: Circumference of a Circle, p. 289: 1, 5, 7, 9, 10

7.6: Area of a Circle, p. 296: 1, 5, 12, 15

I will supply supplementary material on the sector of a circle, recognition of solids, and surface area and volumes of solids.

from course content summary

Apply concepts of sector and arc length of a circle

Recognize various geometric solids such as cylinder, cone, pyramid, prism and sphere.

Calculate surface area and volume of various geometric solids

Use the properties of inscribed and circumscribed polygons and circles to find unknown amount

7.6

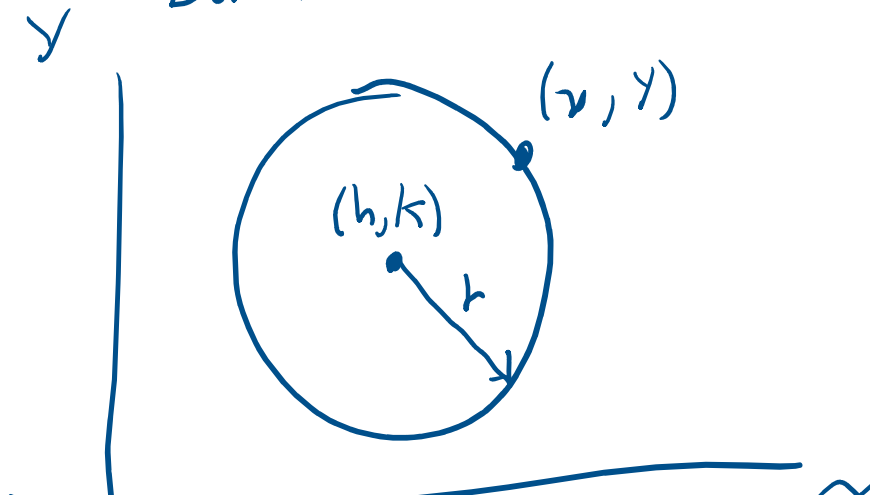
Memorize

THEOREM 1

The area of circle is π times the square of its radius.

$$A = \pi r^2$$

Let A = area of circle with radius r



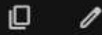


Definition $\pi = \frac{\text{circumference}}{\text{diameter}} = \frac{C}{D}$ memorize

$$\boxed{D = 2r} \text{ memorize}$$

$$\pi = \frac{C}{2r}$$

$$\Rightarrow \boxed{C = 2\pi r} \text{ memorize}$$



Could you please give me a proof of this formula for my Basic Technical Math class. We do not know calculus here.

◆ Analysis ▾

- ▶ To prove the area of a circle without using calculus, we can use a method known since ancient times: the "**Wedge Method**" (or the rearrangement method). This technique transforms the circle into a shape we already know how to measure, like a rectangle or a parallelogram.

The Wedge Method Proof

1. Slice the Circle

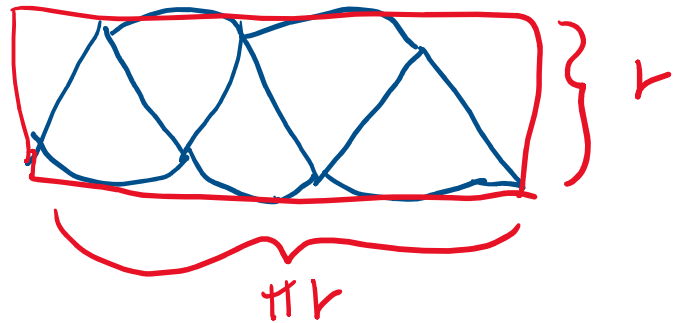
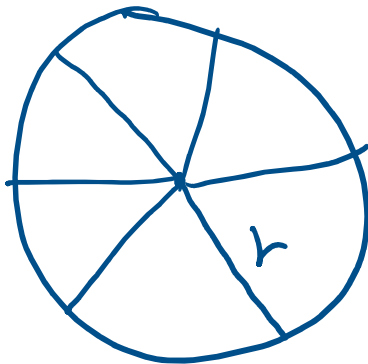
Imagine a circle with radius r and circumference C . We know the circumference is calculated as $C = 2\pi r$. Now, imagine slicing the circle into several equal "pie slices" or sectors.

2. Rearrange the Slices

Take those slices and lay them out in a row, alternating their orientation (one pointing up, the next pointing down). As you can see, the resulting shape looks roughly like a **parallelogram**.

3. Refine the Shape

If we were to slice the circle into more and more slices—say, 100 or 1,000—the top and bottom edges would become straighter, and the side edges would become almost perfectly vertical. Eventually, the shape becomes a **rectangle**.



4. Calculate the Dimensions

To find the area of this new rectangle, we need its base and height:

- **Height:** The height of the rectangle is simply the **radius (r)** of the circle, as that is the length of each slice.
- **Base:** The top and bottom edges of the rectangle are made up of the curved outer edges of the slices. Since half the slices point up and half point down, the base of the rectangle is exactly **half of the circumference**.

$$\text{Base} = \frac{1}{2}C = \frac{1}{2}(2\pi r) = \pi r$$

5. The Final Formula

The area of a rectangle is **base \times height**. Substituting our values:

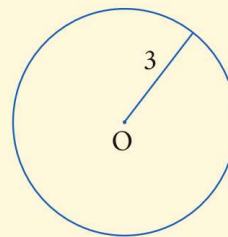
$$\text{Area} = (\pi r) \times r$$

$$\text{Area} = \pi r^2$$

Note for your Technical Math class: This is a classic example of how we can use "exhaustion"—breaking a complex shape into infinite smaller parts—to find a precise formula even without the formal machinery of calculus!

? EXAMPLE A

Find the area of the circle:



SOLUTION

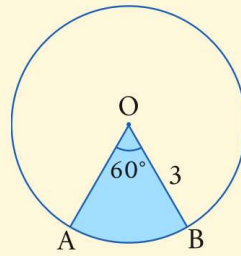
$$A = \pi r^2 = \pi(3)^2 = 9\pi \approx 9(3.14) = 28.26.$$

↑
exact
area

↑
approximate
area

? EXAMPLE B

Find the shaded area:



Let A = area of shaded sector

$$\frac{A}{\text{area of circle}} = \frac{60^\circ}{360^\circ} = \frac{1}{6}$$

$$A = \left(\frac{1}{6}\right) (\text{area of circle})$$

$$A = \left(\frac{1}{6}\right) \pi r^2$$

$$A = \left(\frac{1}{6}\right) (\pi) (3^2)$$

$$A = \frac{9\pi}{6}$$

$$A = \frac{3\pi}{2}$$

memorize

$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{central angle}}{360^\circ}$$

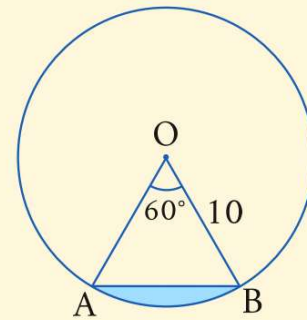
$$\text{Area of sector} = \frac{D}{360} \cdot \text{Area of circle}$$

or simply

$$A = \frac{D}{360} \cdot \pi r^2$$

? EXAMPLE C

Find the shaded area:



SOLUTION

Let us first find the area of triangle OAB (Figure 3).

$\triangle OAB$ is equilateral with base $b = AB = 10$. Drawing in height $h = OC$ we have that $\triangle AOC$ is a $30^\circ - 60^\circ - 90^\circ$ triangle with $AC = 5$ and $h = 5\sqrt{3}$. Therefore

shaded area = area of sector OAB – area of triangle OAB

$$\begin{aligned} &= \frac{D}{360} \pi r^2 - \frac{1}{2} bh \\ &= \frac{60}{360} \pi (10)^2 - \frac{1}{2} (10)(5\sqrt{3}) \\ &= \frac{1}{6} (100\pi) - \frac{1}{2} (50\sqrt{3}) \\ &= \frac{50\pi}{3} - 25\sqrt{3} \end{aligned}$$

$$\begin{aligned} &\approx \textcircled{0} \frac{50(3.14)}{3} - 25(1.732) \\ &= 52.33 - 43.30 = 9.03. \end{aligned}$$

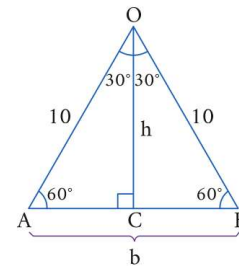


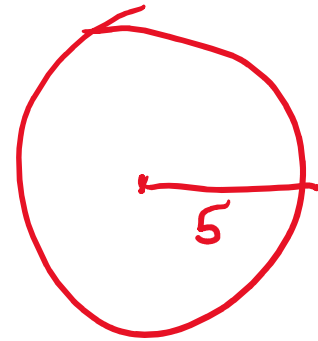
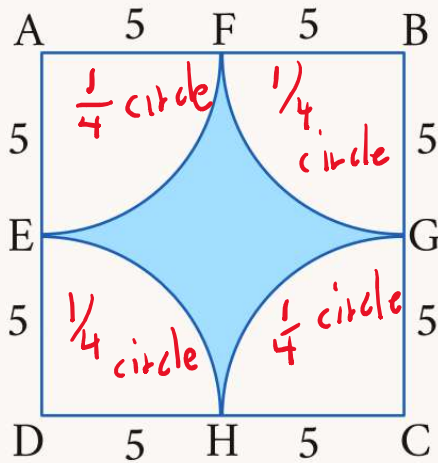
Figure 3.
Triangle OAB with base b and height h .

ANSWER: $\frac{50\pi}{3} - 25\sqrt{3}$ ~~or~~ 9.03.

The shaded area in Example C is called a **segment** of the circle. The area of a segment is obtained by subtracting the area of the triangle from the area of the sector.

15 - 30. Find the shaded area. Answers may be left in terms of π and in radical form.

29.



Area of circle
 $= \pi r^2 = \boxed{25\pi}$

area of square ABCD
 $= 10^2 = 100$

shaded area = area of square
- area of circle

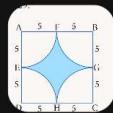
$= \boxed{100 - 25\pi}$

$100 - 25 * \pi = 21.46018366025517$

$100 - 25 * 3.14 = 21.5$

$100 - 25 * 3.14159 = 21.46025$

may be



Just give me one hint to get started.

Analysis

To find the shaded area, think about the **entire square** first.

Notice that the white space in the corners is formed by four identical circular sectors (quarter-circles) centered at points A, B, C, and D. Each of these sectors has a radius of 5.

The Hint:

The shaded area is equal to the **Area of the Square** minus the **Area of one full Circle**.

Google Gemini



What is the arc length of a circle?

The **arc length** is the distance along the curved line that forms a specific section of a circle's circumference. You can think of it as a "fraction" of the total perimeter.

The Logic Behind the Formula

To find the length of an arc, you essentially compare the angle of that arc to the full 360° of a circle.

- **The Full Circumference:** The distance all the way around a circle is $C = 2\pi r$.
- **The Fraction:** If you have an arc with a central angle θ , that arc represents the fraction $\frac{\theta}{360}$ of the entire circle.

not in MTH 111

The Formulas

Depending on whether you are using degrees or radians, the formula looks slightly different:

1. Using Degrees

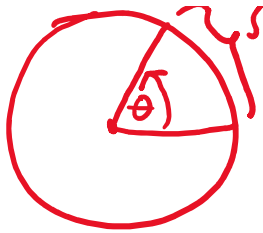
If your angle θ is in degrees:

$\theta = \text{central angle}$

$$S = \text{Arc Length} = \frac{\theta}{360} \times 2\pi r$$



*$\theta = \text{central angle}$
 $s = \text{arc length}$*



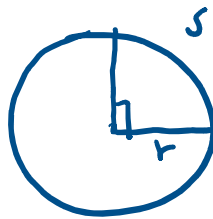
θ = central angle
s = arc length

$$\frac{s}{c} = \frac{\theta}{360} = \frac{s}{2\pi r}$$

A Practical Example

If you have a circle with a radius of 10 cm and you want to find the length of a 90° arc (a quarter-circle):

1. Find the fraction: $90/360 = 1/4$.
2. Calculate full circumference: $2 \times \pi \times 10 = 20\pi$.
3. Multiply: $1/4 \times 20\pi = 5\pi$ (approximately 15.7 cm).



$$r = 10 \text{ cm}$$

$$\frac{s}{2\pi(10 \text{ cm})} = \frac{90^\circ}{360^\circ}$$

$$s = \left(\frac{90}{360}\right)(20\pi) \text{ cm}$$

$$s = \left(\frac{9}{36}\right)(20\pi) \text{ cm}$$

$$s = \left(\frac{1}{4}\right)(20\pi) \text{ cm}$$

$$s = 5\pi \text{ cm} \approx 15.7 \text{ cm}$$

$$5 * \pi = 15.70796326794897$$

Your Name MTH 111 quiz 5 Write each problem. Calculator needed.

1. Write 3046.21 in scientific notation.

$$3.04621 \times 10^3$$

2. Round $\sqrt{456}$ to the nearest hundredth.

$$\text{Sqrt}(456) = 21.35415650406262$$

21.35

3. Is $x = 3$ a solution of $3x - 5 = 4$?

Answer in two different ways.

check

$$3(3) - 5 \stackrel{?}{=} 4$$

$$9 - 5 \stackrel{?}{=} 4$$

$$4 = 4 \checkmark$$

$\therefore x = 3$ is a solution

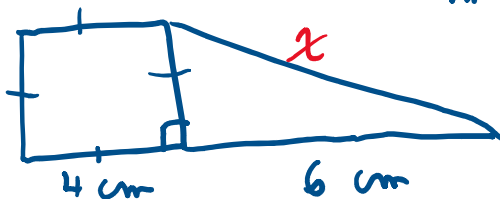
solve the equation

$$3x = 9$$

$$x = 3$$

$\therefore x = 3$ is a solution, in fact it is the solution

4.



Find the area and perimeter of the figure.

area = area of \square + area of Δ

$$\text{area of square} = (4 \text{ cm})^2 = 16 \text{ cm}^2$$

$$\text{area of } \Delta = \left(\frac{1}{2}\right) (\text{base}) (\text{height})$$

$$= \left(\frac{1}{2}\right) (6 \text{ cm}) (4 \text{ cm}) = 12 \text{ cm}^2$$

$$\text{total area} = 16 \text{ cm}^2 + 12 \text{ cm}^2 = 28 \text{ cm}^2$$

$$\text{Perimeter} = 3(4 \text{ cm}) + 6 \text{ cm} + x$$

$$= 18 \text{ cm} + x$$

$$\text{Pyth Thm} \Rightarrow x = \sqrt{(6 \text{ cm})^2 + (4 \text{ cm})^2}$$

$$= \sqrt{36 + 16} \text{ cm}^2$$

$$= \sqrt{36+16} \text{ cm}$$

$$= \sqrt{52} \text{ cm}$$

$$= \sqrt{4 \cdot 13} \text{ cm}$$

$$\boxed{x = 2\sqrt{13}} \text{ cm}$$

$$\text{Perimeter} = (18 + 2\sqrt{13}) \text{ cm} \approx \boxed{25.2 \text{ cm}}$$

$$18 + 2 \cdot \sqrt{13} = 25.21110255092798$$

5. Find the x-intercept and y-intercept of the line given by $5x - 8y = 10$.

x-intercept

$$5x - 8(0) = 10$$

$$5x = 10$$

$$\boxed{x = 2}$$

or the point $(2, 0)$

y-intercept

$$5(0) - 8y = 10$$

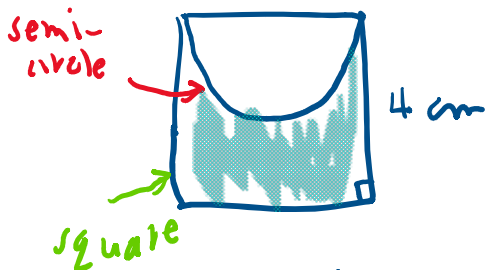
$$-8y = 10$$

$$y = -\frac{10}{8}$$

$$\boxed{y = -\frac{5}{4}}$$

or the point $(0, -\frac{5}{4})$

6. Find the shaded area. Give exact answer. Then, round to nearest hundredth.



Let $A =$ shaded area

$A =$ area of square - area of semicircle

$$= 16 \text{ cm}^2 - \frac{\pi (2 \text{ cm})^2}{2}$$

$$= \left(16 - \frac{\pi 4}{2}\right) \text{ cm}^2$$

$$\boxed{A = (16 - 2\pi) \text{ cm}^2 \approx 9.72 \text{ cm}^2}$$

$$16 - 2 \cdot \pi = 9.716814692820413$$

7. Ask and answer an "essay" question. Your answer should show understanding of a concept. No calculation. Answer in one or two sentences.