6. Health Option

6.1 Measurement; Health Applications
   6.1 Exercise Set, page 663 (645): 1, 3, 5, 8, 14, 17, 19,
6.2 Ratio, rate, and percent; Health Applications
   6.2 Exercise Set, page 680 (662): 1, 6, 10, 11, 18, 20, 21

I will provide supplementary material about computing values in direct, indirect, and inverse variation.

Elementary College Geometry - 2021 ed. Henry Africk
Chapter 1 - Lines, Angles, and Triangles
1.6 Triangle Classifications, p. 62: 1, 3, 9

Chapter 6 - Area and Perimeter
6.1 The Area of a Rectangle and Square, p.213: 1, 4, 7, 15, 21

6.3 The Area of a Triangle, p. 225: 1, 7, 21

memorize

Definition 4.5. Suppose x, y and z are variable quantities. We say

- y **varies directly with** (or is **directly proportional to**) x if there is a constant k such that $y = kx$.
- y **varies inversely with** (or is **inversely proportional to**) x if there is a constant k such that $y = \frac{k}{x}$.
- z **varies jointly with** (or is **jointly proportional to**) x and y if there is a constant k such that $z = kxy$.

The constant k in the above definitions is called the **constant of proportionality**.

Example 4.3.6. Translate the following into mathematical equations using Definition 4.5.

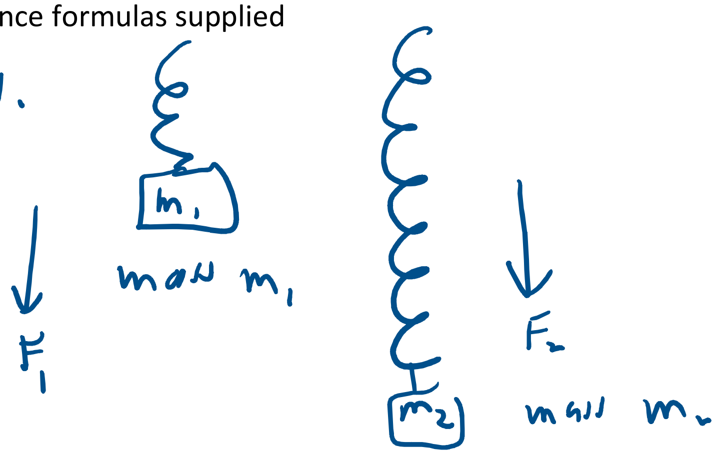
1. Hooke's Law: The force F exerted on a spring is directly proportional the extension x of the spring.
   $F = kx$
2. Boyle's Law: At a constant temperature, the pressure P of an ideal gas is inversely proportional to its volume V.
   $P = \frac{k}{V} \iff PV = k$
3. The volume V of a right circular cone varies jointly with the height h of the cone and the square of the radius r of the base.
4. Ohm's Law: The current I through a conductor between two points is directly proportional to the voltage V between the two points and inversely proportional to the resistance R between the two points.

science formulas supplied

1 6 6

science formulas supplied

1.



Homework

To help you practice translating these mathematical definitions into algebraic expressions, here are several well-known laws from physics and economics described in sentence form.

Direct Variation ($y = kx$)

- **Hooke's Law:** The force needed to extend or compress a spring by some distance is directly proportional to that distance.
- **Charles's Law:** For a fixed mass of an ideal gas at constant pressure, the volume of the gas varies directly with its absolute temperature.
- **Ohm's Law:** The current through a conductor between two points is directly proportional to the voltage across the two points (assuming temperature and other physical conditions remain constant).

Inverse Variation ($y = \frac{k}{x}$)

- **Boyle's Law:** For a fixed amount of an ideal gas kept at a fixed temperature, the pressure of the gas varies inversely with its volume.
- **Newton's Law of Universal Gravitation (Distance):** The gravitational force between two objects varies inversely with the square of the distance between their centers.
- **The Law of Demand:** In a simplified market model, the quantity demanded of a good varies inversely with its price, provided all other factors remain equal.

Joint Variation ($z = kxy$)

- **Newton's Second Law of Motion:** The net force acting upon an object is jointly proportional to the mass of the object and its acceleration.
- **The Ideal Gas Law (Relationship):** The pressure of an ideal gas varies jointly with the number of moles of the gas and its absolute temperature (when volume is held constant).
- **The Area of a Triangle:** The area of a triangle varies jointly with the length of its base and its height.
- **Simple Interest:** The interest earned on an investment varies jointly with the principal amount invested and the time for which the money is invested (at a fixed rate).

A Note on Combined Variation

Sometimes, laws use a combination of these rules. For example, **Newton's Law of Gravitation** states that the force varies **jointly** with the masses of the two objects and **inversely** with the square of the distance between them.

1.6
Memorize

Triangles may be classified according to the relative lengths of their sides:

An **equilateral** triangle has three equal sides.

An **isosceles** triangle has at least two equal sides.

A **scalene** triangle has no equal sides.

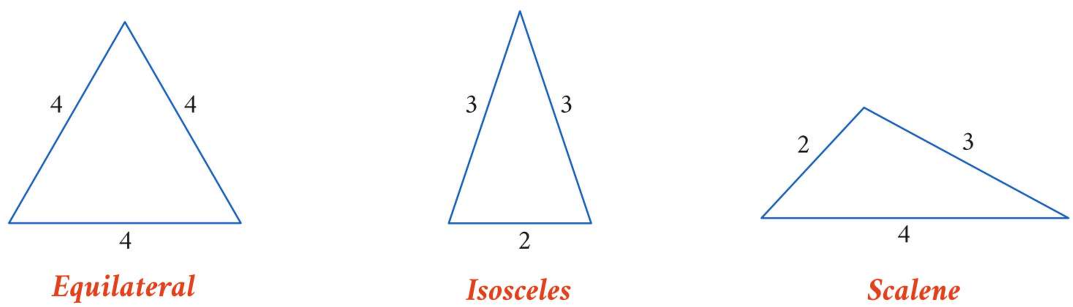


Figure 1. Triangles classified according to their sides.

response

Can an equilateral triangle also be isosceles?

That's a great question, and it often trips people up because we tend to think of shapes in very specific "boxes." The short answer is: **Yes, absolutely.**

In fact, **every equilateral triangle is also an isosceles triangle.**

Here is why:

- **The Definition of Isosceles:** By the standard geometric definition, a triangle is isosceles if it has **at least two** equal sides.
- **The Definition of Equilateral:** A triangle is equilateral if **all three** of its sides are equal.

Since an equilateral triangle has three equal sides, it automatically satisfies the requirement of having at least two. Think of it like a "special club": to get into the Isosceles Club, you only need two matching sides. The Equilateral Club is more exclusive (you need three), but once you're in the Equilateral Club, you've already met the requirements for the Isosceles one!

memorize

Triangles may also be classified according to the measure of their angles:
An **acute** triangle is a triangle with three acute angles.
An **obtuse** triangle is a triangle with one obtuse angle.
An **equiangular** triangle is a triangle with three equal angles.
Each angle of an equiangular trialngle must be 60°. We will show in section 2.5 that equiangular triangles are the same as equilateral triangles.

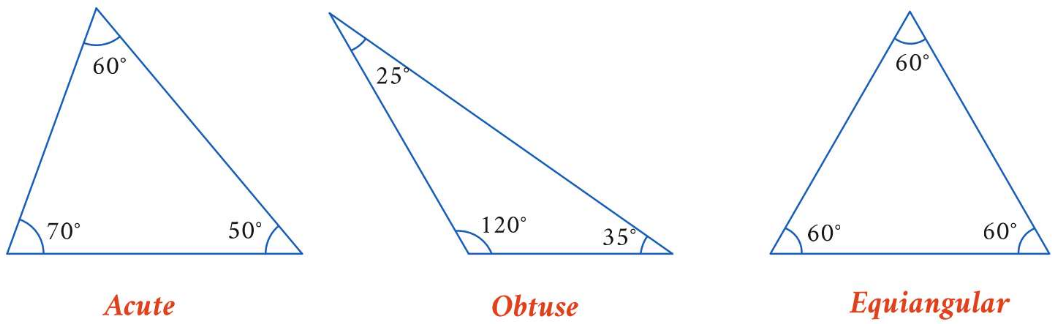
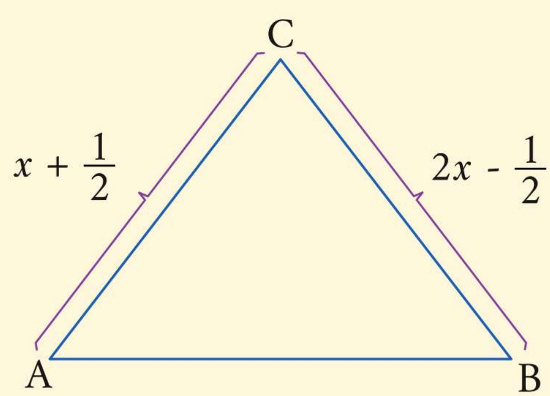


Figure 2. Triangles classified according to their angles.

Note: a triangle is equilateral if and only if it is equiangular.

? EXAMPLE A

Find x if △ABC is isosceles with AC = BC:



$$x + \frac{1}{2} = 2x - \frac{1}{2}$$

$$\frac{1}{2} + \frac{1}{2} = x$$

$$\boxed{x = 1}$$

check $1 + \frac{1}{2} \stackrel{?}{=} 2(1) - \frac{1}{2}$

$1\frac{1}{2} = 1\frac{1}{2}$ ✓

check

$1\frac{1}{2} = 1\frac{1}{2}$ ✓

memorize

height

An **altitude** of a triangle is a line segment from a vertex perpendicular to the opposite side. In Figure 4, CD and GH are altitudes. Note that altitude GH lies outside △EFG and side EF must be extended to meet it.

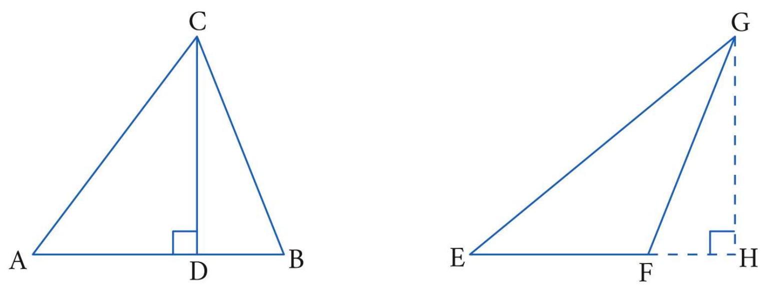


Figure 4. CD and GH are altitudes.

Memorize

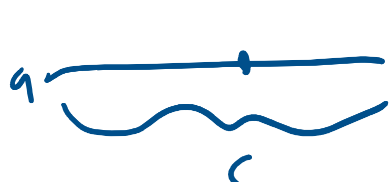
THEOREM 1

The sum of any two sides of a triangle is greater than the remaining side.

Definition: a theorem is a proven mathematical statement.

$a+b>c$

↓ collapse △



$a+b=c$

"degenerate triangle"

"degenerate triangle"

$\angle a = 0$

$\angle b = 0$

$\angle c = 180^\circ$

sum $= 180^\circ$

Your Name MTH 111 quiz 4 write each problem.

1. Convert 0.75 to a percent.

$0.75 = \frac{75}{100} = 75\left(\frac{1}{100}\right) = \boxed{75\%}$

2. Convert $\frac{5}{8}$ to a decimal.

.

$\boxed{0.625}$

$$\begin{array}{r} .625 \\ 8\overline{)5.000} \\ \underline{48} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

5/8=0.625

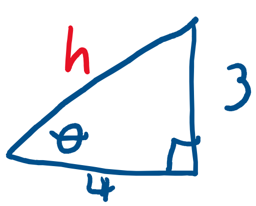
3. A popular fast food burger weighs 7.5 ounces and contains 540 calories, 29 grams of fat, 43 grams of carbohydrates, and 25 grams of protein. Find the unit rate of:

a. calories per ounce

$$\frac{540 \text{ cal}}{7.5 \text{ oz}} = \frac{\frac{540}{7.5} \text{ cal}}{1 \text{ oz}} = \boxed{72 \frac{\text{cal}}{\text{oz}}}$$

540/7.5=72

4.



Find $\sin(\theta)$

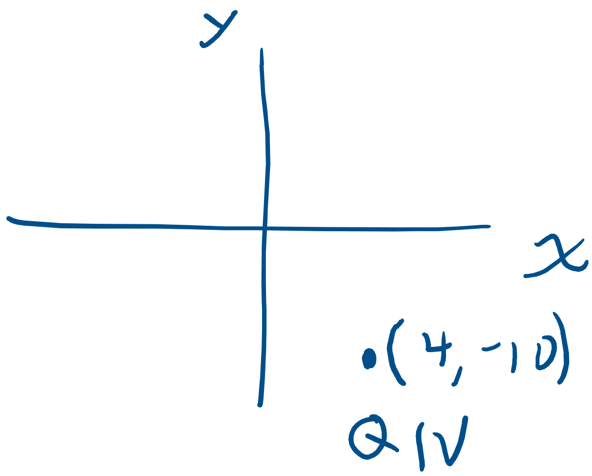
$\sin(\theta) = \frac{\text{opp}}{\text{hyp}} = \boxed{\frac{3}{5}}$

Let h = hypotenuse

Let h = hypotenuse

$h = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = \boxed{5 = h}$

5. Plot the point $(4, -10)$ in the rectangular coordinate system and identify the quadrant it is in.



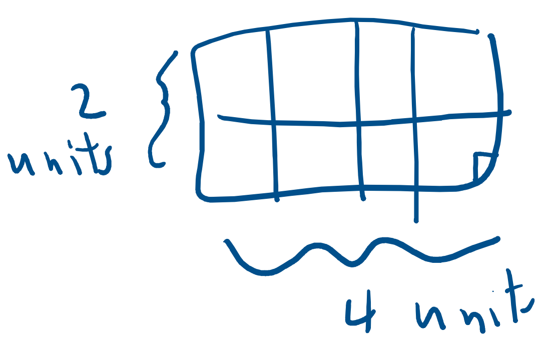
6.1 THE AREA OF A RECTANGLE AND SQUARE

Memorize

THEOREM 1

The area of a rectangle is the length times its width.

$$\boxed{A = lw}$$



area $= (2)(4)$ unit^2

$= 8$ unit^2

Memorize

THEOREM 2

The area of a square is the square of one of its sides.

$$\boxed{A = s^2}$$

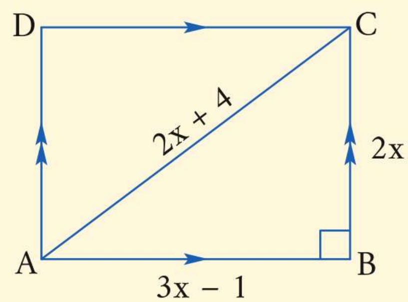
Let s = length of side of the square

Memorize

The **perimeter** of a polygon is the sum of the lengths of its sides.

EXAMPLE C

Find the area and perimeter of rectangle ABCD:



Strategy: find x

$(3x-1)^2 + (2x)^2 = (2x+4)^2$ Pyth thm

$(9x^2 - 6x + 1) + 4x^2 = 4x^2 + 16x + 16$

$13x^2 - 6x + 1 = 4x^2 + 16x + 16$

$(13x^2 - 4x^2) + (-6x - 16x) + 1 - 16 = 0$

$9x^2 - 22x - 15 = 0$ quadratic equation

$$x = \frac{22 \pm \sqrt{22^2 - (4)(9)(-15)}}{18}$$

$$x = \frac{22 \pm \sqrt{484 + (60)(9)}}{18}$$

$$\overline{\qquad\qquad}$$
$$18$$

$$x = \frac{22 \pm \sqrt{484 + 540}}{18}$$

$$x = \frac{22 \pm \sqrt{1024}}{18}$$

$$x > 0$$

$$\Rightarrow x = \frac{22 + \sqrt{1024}}{18}$$

Sqrt(1024)=32

$$x = \frac{22+32}{18} = \frac{54}{18}$$

$$\boxed{x = 3}$$

$$\text{Perimeter} = 2(2x + 3x - 1)$$
$$= 2(5x - 1)$$
$$= 2(5(3) - 1)$$
$$= 2(15 - 1)$$
$$= 2(14)$$

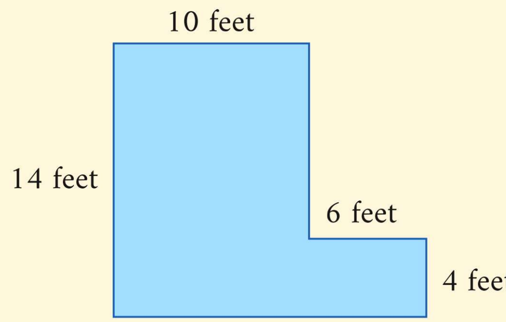
$$\boxed{\text{Perimeter} = 28}$$

$$\text{area} = 2x(3x - 1)$$
$$= 2(3)((3)(3) - 1)$$
$$= 6(9 - 1)$$
$$= 6(8)$$

$$\boxed{\text{area} = 48}$$

? EXAMPLE E

An L-shaped room has the dimensions indicated in the diagram. How many one by one foot tiles are needed to tile the floor?



Find the area of the room.

1.3
memorize

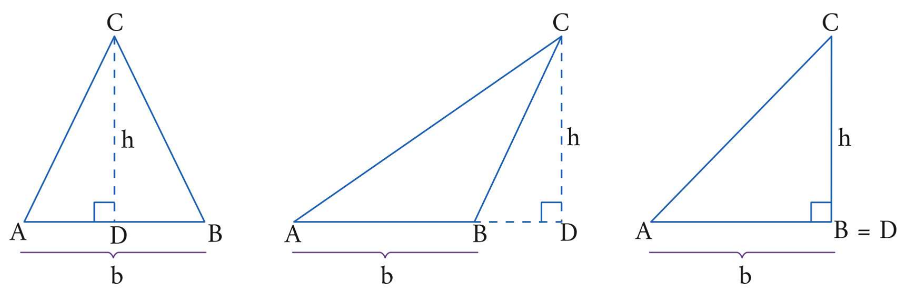


Figure 1. Triangles with base *b* and height *h*.

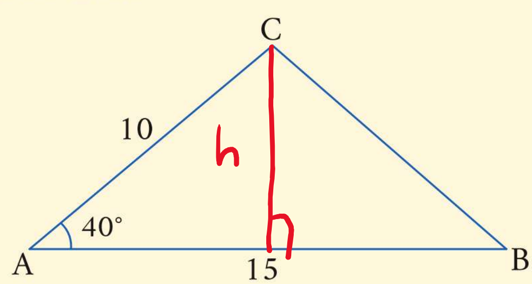
THEOREM 1

The area of a triangle is equal to one-half of its base times its height.

$$\boxed{A = \frac{1}{2}bh}$$

? EXAMPLE B

Find the area to the nearest tenth:



Let h = height of $\triangle ABC$

$$\sin 40^\circ = \frac{opp}{hyp} = \frac{h}{10}$$

$\sin 40^\circ = \frac{opp}{hyp} = \frac{h}{10}$

$\boxed{h = 10 \sin 40^\circ}$

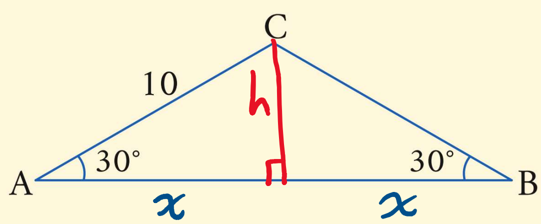
```
10*sin(40)
             6.427876097
ans*15/2
             48.20907073
```

$Area \approx \left(\frac{1}{2}\right)(15)(6.4)$

$\boxed{Area \approx 48.2}$

? EXAMPLE D

Find the area and perimeter:



$\sin(30^\circ) = \frac{opp}{hyp} = \frac{h}{10}$

$h = 10 \sin(30^\circ) = (10)\left(\frac{1}{2}\right) = 5$

$\boxed{h = 5}$

$\frac{x}{10} = \frac{adj}{hyp} = \cos(30^\circ)$

$x = 10 \cos(30^\circ) = 10\left(\frac{\sqrt{3}}{2}\right) = 5\sqrt{3}$

$Area = \frac{1}{2}(base)(height) = \left(\frac{1}{2}\right)(2x)5 = 5x$

Area = $\frac{1}{2}$(base)(height) = $\left(\frac{1}{2}\right)(2x)\,5 = 5x$

Area = $5(5\sqrt{3}) = \boxed{25\sqrt{3}}$

Perimeter $= 2x + 20$

$= 2(5\sqrt{3}) + 20$

Perimeter $= 10\sqrt{3} + 20$

Area = $\frac{1}{2}$ bh = $\frac{1}{2}(5\sqrt{3} + 5\sqrt{3})(5) = \frac{1}{2}(10\sqrt{3})(5) = \frac{1}{2}(50\sqrt{3}) = 25\sqrt{3}$.

Perimeter = $10 + 10 + 5\sqrt{3} + 5\sqrt{3} = 20 + 10\sqrt{3}$.

ANSWER: $A = 25\sqrt{3}$, $P = 20 + 10\sqrt{3}$.