

## 1.2 Integers

1.2 Exercise Set, page 69 (51): 1, 2, 6, 18, 25, 32, 39, 40, 48

## 1.3 Fractions

1.3 Exercise Set, page 90 (72): 5, 8, 20, 27, 44, 49, 56

## 1.4 Decimals

1.4 Exercise Set, page 112 (94): 1, 2, 4, 5, 9, 10, 18, 19, 25, 31, 33, 42, 44

## 1.5 Exponents and Scientific Notation

1.5 Exercise Set, page 142 (124): 1, 3, 6, 8, 11, 18, 20, 22, 26, 28, 30, 32, 34, 36, 38, 41

## 1.6 Roots and Radicals

1.6 Exercise Set, page 163 (145): 1, 3, 12, 14, 15, 17, 19

I will provide supplementary material about identifying significant digits.

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Your Name MTH 111 quiz 1

Open homework notebook. Closed everything else. No calculator.

1.2: 1a, 2a

1.3: 5

1.4: 1

In the following exercises, order each of the following pairs of numbers, using &lt; or &gt;.

1. a.  $9 > 4$ 

In the following exercises, simplify.

2. a.  $|-32| = 32$

## 1.3 Exercise Set

In the following exercises, multiply.

5.  $\left(-\frac{63}{84}\right) \cdot \left(-\frac{44}{90}\right)$

$$= \frac{\cancel{3}(\cancel{9})(\cancel{4})(11)}{\cancel{4}(\cancel{2})(\cancel{9})(10)}$$

$$= \frac{11}{30}$$

$$(-63/84)*(-44/90)=0.3667$$

$$11/30=0.3667$$

1.4

In the following exercises, round each number to the nearest tenth.

1.  $0.\underline{6}7$   
 $\uparrow$   
 $0.7$

1.2: 40

In the following exercises, simplify each expression.

40.  $-4^2$

$= -(4^2)$   
 $= -16$

$\xrightarrow{\hspace{10em}}$   
 $(-4)^2 = 16$

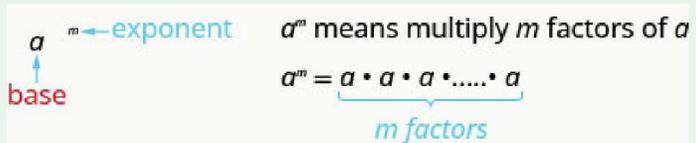
PEMDAS  
 parentheses  
 exponents  
 mult  
 div  
 add  
 sub

left to right

1.5

Memorize

Exponential Notation (Power)



a raised to the mth power

$3^4 = 3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$

EXAMPLE 1

Simplify: a)  $4^3$  b)  $7^1$  c)  $\left(\frac{5}{6}\right)^2$  d)  $(0.63)^2$ .

**Solution**

a)	$4^3$
Multiply three factors of 4.	$4 \cdot 4 \cdot 4$
Simplify.	64
b)	$7^1$ ✓
Multiply one factor of 7. <i>does not make sense</i>	7 ✓

Memorize

Product Property for Exponents

If  $a$  is a real number, and  $m$  and  $n$  are counting numbers, then

$$a^m \cdot a^n = a^{m+n}$$

$$\begin{aligned} 2^3 \cdot 2^2 &= (2 \cdot 2 \cdot 2)(2 \cdot 2) \\ &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \\ &= 2^5 = 2^{3+2} \end{aligned}$$

Memorize

Quotient Property for Exponents

If  $a$  is a real number,  $a \neq 0$ , and  $m$  and  $n$  are whole numbers, then

$$\frac{a^m}{a^n} = a^{m-n}, m > n \text{ and } \frac{a^m}{a^n} = \frac{1}{a^{n-m}}, n > m$$

$$-5 \quad \checkmark \quad -1 \quad - \quad \dots \quad 2 \quad \dots$$

$$\frac{2^5}{2^2} = \frac{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2 \cdot 2}{\cancel{2} \cdot \cancel{2}} = 2^3 = 2^{5-2}$$

$$\frac{2^3}{2^3} = \frac{8}{8} = 1$$

by our rule  $\frac{2^3}{2^3} = 2^{3-3} = 2^0$

To keep our beautiful rule

define  $2^0 = 1$

memorize  $a^0 = 1$   $a \neq 0$

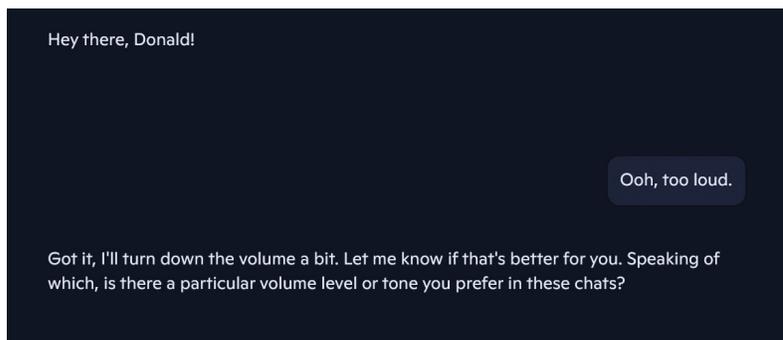
$$3^2 = 9$$

$$0^0 = 1$$



Scientific Notebook

$$0^0 = 1$$



Hey there, Donald!

Ooh, too loud.

Got it, I'll turn down the volume a bit. Let me know if that's better for you. Speaking of which, is there a particular volume level or tone you prefer in these chats?

Thank you.

Okay, Co-Pilot, what is zero raised to the zeroth power?

Ah, the classic zero to the zeroth power question! Mathematicians love to debate this one. It's kind of like a gray area because depending on the context, it can be considered indeterminate. Some say it's one, others say it doesn't have a value. So what's your take? Do you lean one way or the other on the whole zero to the zeroth power debate?

$$\frac{2^3}{2^5} = \frac{8}{32} = \frac{1}{4} = \frac{1}{2^2}$$

rule  $\frac{2^3}{2^5} = 2^{3-5} = 2^{-2}$

define  $2^{-2} = \frac{1}{2^2}$

memorize

$$a^{-n} = \frac{1}{a^n}$$

$$\frac{2^5}{2^4} = \frac{32}{16} = 2$$

5            4            1

$$2^1 \quad 1+0$$

rule  $\frac{2^5}{2^4} = 2^{5-4} = 2^1$

define  $2^1 = 2$

memorize  $a^1 = a$

rule  $2^{\frac{1}{2}} 2^{\frac{1}{2}} = 2^{\frac{1}{2} + \frac{1}{2}} = 2^1 = 2$

$$\sqrt{2} \sqrt{2} = 2$$

we define  $2^{\frac{1}{2}} = \sqrt{2}$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

memorize  $a^{\frac{1}{n}} = \sqrt[n]{a}$

$$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$$

$$8 \wedge (1/3)$$

$$8^{(1/3)}=2$$

EXAMPLE 3

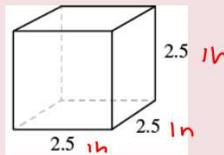
A cube is 2.5 inches on each side. Find its a) volume and b) surface area.

**Solution**

Step 1 is the same for both a) and b), so we will show it just once.

110 1. Operations with Real Numbers

Step 1. **Read** the problem. Draw the figure and label it with the given information.



a)	
Step 2. <b>Identify</b> what you are looking for.	the volume of the cube
Step 3. <b>Name.</b> Choose a variable to represent it.	let $V$ = volume
Step 4. <b>Translate.</b> Write the appropriate formula.	$V = s^3$
Step 5. <b>Solve.</b> Substitute and solve.	$V = (2.5)^3$ $V = 15.625$ $V = (2.5 \text{ in})^3$ $= (2.5)^3 \text{ in}^3$ $V = 15.625 \text{ in}^3$
Step 6. <b>Check:</b> Check your work.	
Step 7. <b>Answer</b> the question.	The volume is 15.625 cubic inches.

b)	
Step 2. <b>Identify</b> what you are looking for.	the surface area of the cube
Step 3. <b>Name.</b> Choose a variable to represent it.	let $S$ = surface area
Step 4. <b>Translate.</b> Write the appropriate formula.	$S = 6s^2 = 6(2.5 \text{ in})^2$
Step 5. <b>Solve.</b> Substitute and solve.	$S = 6 \cdot (2.5)^2$ $S = 37.5$ $= 6 \cdot (2.5)^2 \text{ in}^2$ $= 37.5 \text{ in}^2$
Step 6. <b>Check:</b> The check is left to you.	
Step 7. <b>Answer</b> the question.	The surface area is 37.5 square inches.

$2.5^3 = 15.625$

Memorize

## Scientific Notation

A number is expressed in scientific notation when it is of the form  $a \times 10^n$  where  $1 \leq a < 10$  and  $n$  is an integer

$$365 = 3.65 \times 10^2$$

2 places

$$1 \leq 3.65 < 9$$

$3.65 = a$   
 $2 = n$

$$1,000,000 = 1 \times 10^6$$

$a = 1$   
 $n = 6$

---

$$0.0038 = 3.8 \times 10^{-3}$$

$a = 3.8$   
 $n = -3$

Memorize the process

### HOW TO: Convert from decimal notation to scientific notation

1. Move the decimal point so that the first factor is greater than or equal to 1 but less than 10.
2. Count the number of decimal places,  $n$ , that the decimal point was moved.
3. Write the number as a product with a power of 10.  
If the original number is:

- greater than 1, the power of 10 will be  $10^n$ .
- between 0 and 1, the power of 10 will be  $10^{-n}$ .
- Check.

Memorize the process

**HOW TO: Convert scientific notation to decimal form.**

To convert scientific notation to decimal form:

1. Determine the exponent,  $n$ , on the factor 10.
2. Move the decimal  $n$  places, adding zeros if needed.
  - If the exponent is positive, move the decimal point  $n$  places to the right.
  - If the exponent is negative, move the decimal point  $|n|$  places to the left.
3. Check.

### 1.5 Exercise Set

In the following exercises, simplify each expression with exponents.

c.  $\left(\frac{2}{5}\right)^3$

$= \frac{2^3}{5^3} = \frac{8}{125}$

d.  $(0.2)^4$

$(0.2)^2 (0.2)^2$   
 $(.04) (.04)$

$.0016$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$(2/5)^3 = 0.064$  One-Note converted to a decimal

$(2/5)^3$	$\frac{8}{125}$
$\frac{8}{125}$	0.064

10.

a.  $\frac{1}{5^{-2}}$

$= \frac{1}{\frac{1}{5^2}}$

invert denominator and multiply

$$= (1) \left( \frac{5^2}{1} \right)$$

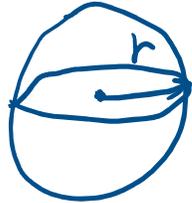
$$= 5^2 = \boxed{25}$$

and multiply

$$a^{-n} = \frac{1}{a^n}$$

25. **Golf ball** A golf ball has a radius of 4.5 centimetres. Find its a) volume and b) surface area.

supplied



$V =$  volume of sphere,  $r =$  radius

$$V = \frac{4\pi r^3}{3}$$

round to nearest tenth  $\text{cm}^3$

$4\pi(4.5)^3/3$	$\frac{243 \cdot \pi}{2}$
$\frac{243 \cdot \pi}{2}$	381.7035074

$$V = \left( \frac{4}{3} \right) \pi (4.5 \text{ cm})^3$$

$$= \frac{4\pi(4.5)^3}{3} \text{ cm}^3$$

$$\approx \boxed{381.7 \text{ cm}^3}$$

The volume of the sphere is about  $381.7 \text{ cm}^3$

39. The probability of winning the lottery was about 0.0000000057. Write the number in scientific notation.

$$\boxed{5.7 \times 10^{-9}} = 57 \times 10^{-10}$$

How do I say this number in words, without the exponent.

If you want to say **0.000000057** in plain English *without* using scientific notation, the cleanest phrasing is:

**“fifty-seven billionths.”**

Here’s why:

- The first nonzero digits are **57**.
- They sit in the **billionths** place (the 8th place to the right of the decimal).

If you want the fully expanded, ultra-literal version, you could also say:

**“zero point zero zero zero zero zero zero zero zero five seven.”**

But “fifty-seven billionths” is the standard mathematical wording.

If you want, I can also show how to name any decimal like this in a systematic way.

Is  $10^{10}$  always 1 billion?

Great question — and the answer is **no**,  $10^{10}$  is *not* always “1 billion.”

Here’s the key:

**us In the United States (and most of the modern world):**

- $10^9 = 1$  billion
- $10^{10} = 10$  billion

**GBEU In older British usage (pre-1970s):**

- “billion” used to mean  $10^{12}$
- So  $10^{10}$  wouldn’t have been called a billion at all.

Today, almost every country uses the **short scale**, where:

- million =  $10^6$
- billion =  $10^9$
- trillion =  $10^{12}$

So in modern math and science, the correct statement is:

**$10^{10} = \text{ten billion.}$**

If you want, I can also give you a quick table of powers of 10 and their names — it’s a nice thing to have handy when teaching.

Wait! In your answer you used  $10^{(-9)}$  as billionth.

Ah — great catch, and I see exactly why it feels inconsistent at first glance. Let’s untangle it cleanly.

### ✓ **Two different things:**

There’s a **billion**  
...and a **billionth**.

They are *not* opposites of each other in the way people sometimes expect.

## 1. "Billion" (a large number)

In the modern **short scale** (used in the U.S. and almost everywhere today):

- **1 billion** =  $10^9$
- So
  - $10^9$  = one billion
  - $10^{10}$  = ten billion

That part is straightforward.

## 2. "Billionth" (a tiny fraction)

A *billionth* is the reciprocal of a billion:

$$\frac{1}{10^9} = 10^{-9}$$

So:

- **1 billionth** =  $10^{-9}$
- **57 billionths** =  $57 \times 10^{-9} = 5.7 \times 10^{-8}$   
(which matches your homework problem)

Copilot miscounted the number of zeros.

1.6

Memorize

Square of a Number

If  $n^2 = m$ , then  $m$  is the **square** of  $n$ .



Memorize

Square Root of a Number

If  $n^2 = m$ , then  $n$  is a **square root** of  $m$ .

Memorize

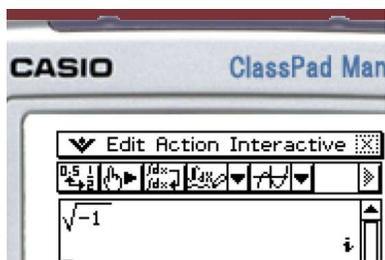
## Square Root Notation

$\sqrt{m}$  is read “the square root of  $m$ ”

radical sign  $\rightarrow \sqrt{m} \leftarrow$  radicand

If  $m = n^2$ , then  $\sqrt{m} = n$ , for  $n \geq 0$ .

The square root of  $m$ ,  $\sqrt{m}$ , is the positive number whose square is  $m$ .



$i$  is an imaginary number with  $i^2 = -1$  not used in MTH 111

## Memorize

### $n^{\text{th}}$ Root of a Number

If  $b^n = a$ , then  $b$  is an  $n^{\text{th}}$  root of  $a$ .  
The principal  $n^{\text{th}}$  root of  $a$  is written  $\sqrt[n]{a}$ .  
 $n$  is called the index of the radical.

## memorize

### Properties of $\sqrt[n]{a}$

When  $n$  is an even number and

- $a \geq 0$ , then  $\sqrt[n]{a}$  is a real number.
- $a < 0$ , then  $\sqrt[n]{a}$  is not a real number.

When  $n$  is an odd number,  $\sqrt[n]{a}$  is a real number for all values of  $a$ .

Be able to do this

Estimate each root between two consecutive whole numbers

$$\sqrt{12}$$

$$1^2 = 1 \quad \text{too small}$$

$$2^2 = 4 \quad \parallel$$

$$3^2 = 9 \quad \parallel$$

$$(\sqrt{12})^2 = 12$$

$$4^2 = 16 \quad \text{too big}$$

$$3 < \sqrt{12} < 4$$

$$\text{Sqrt}(12) = 3.464101615137754$$

$$n < \sqrt{80} < n+1$$

$$8^2 = 64 \quad \text{too small}$$

$$9^2 = 81 \quad \text{too big}$$

$$10^2 = 100 \quad \text{too big}$$

$$8 < \sqrt{80} < 9$$

$$\text{Sqrt}(80) = 8.944271909999159$$

$$1 < \sqrt{2} < 4$$

$$\begin{array}{r}
 1.4142 \\
 \hline
 \sqrt{2.00000000} \\
 \underline{1} \phantom{00} \\
 24 \phantom{00} \\
 \underline{1} \phantom{00} \\
 281 \phantom{00} \\
 \underline{1} \phantom{00} \\
 2824 \phantom{00} \\
 \underline{1} \phantom{00} \\
 28282 \phantom{00} \\
 \underline{1} \phantom{00} \\
 100 \\
 \underline{96} \\
 400 \\
 \underline{281} \\
 11900 \\
 \underline{11296} \\
 61400 \\
 \underline{56564} \\
 4836
 \end{array}$$

Sqrt(2)=1.414213562373095

$$\begin{array}{r}
 5 \\
 \sqrt{11} \\
 \underline{10} \\
 1
 \end{array}$$

$$\begin{array}{r}
 52856 \\
 \hline
 9
 \end{array}$$

(8) = remainder

$$14596$$

(7)

$$36$$

(0)  
1.)

50

U

13

(4)

casting out 9's

72

(0)