11-14-25 MTH 111

CK-12 Interactive Geometry

CK-12 Interactive Geometry | CK-12 Foundation (ck12.org)

1. Basics of Geometry

Before class notes

convert 3 ft into inches

$$12\text{ in} = 1\text{ ft} \qquad \frac{12\text{ in}}{1\text{ ft}} = \frac{1\text{ ft}}{1\text{ ft}}$$

$$\frac{12\text{ in}}{12\text{ in}} = \frac{1\text{ ft}}{12\text{ in}} \qquad \frac{12\text{ in}}{\text{ft}} = 1$$

$$1 = \frac{1\text{ ft}}{12\text{ in}}$$

$$3\text{ ft} = (3\text{ ft})(1) = (3\cancel{\text{ft}})\left(\frac{12\text{ in}}{\cancel{\text{ft}}}\right)$$

$$= \boxed{36\text{ in}}$$ correct

wrong method

$$3\text{ ft} = 3\text{ ft}(1) = 3\text{ ft}\left(\frac{1\text{ ft}}{12\text{ in}}\right) = \frac{3\text{ ft}^2}{12\text{ in}}$$

correct but useless

convert 1 km to cm

$$100\text{ cm} = 1\text{ m}$$

$$1000\text{ m} = 1\text{ km} \Rightarrow \frac{1000\text{ m}}{1\text{ km}} = 1$$

$$1\text{ km} = (1\text{ km})(1)$$

$$= (1\text{ k}\cancel{\text{m}})\left(\frac{1000\text{ m}}{1\text{ k}\cancel{\text{m}}}\right)$$

$$= 1000\text{ m}$$

$$= (1000\text{ m})(1) = (1000\cancel{\text{m}})\left(\frac{100\text{ cm}}{1\cancel{\text{m}}}\right)$$

$$= \boxed{100{,}000\text{ cm}}$$

$$= \boxed{100{,}000\ cm}$$

We could combine these steps

$$1\ km = (1\ \cancel{km})\left(\frac{1000\ \cancel{m}}{1\ \cancel{km}}\right)\left(\frac{100\ cm}{1\ \cancel{m}}\right)$$

$$= \boxed{100{,}000\ cm}$$

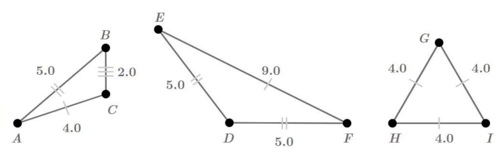
Geometry 1.4
memorize

What is a Triangle?

A triangle is any closed figure made by three non-parallel line segments. Triangles can be classified by their sides and by their angles.

Triangle Classification Based on Sides

When classifying a triangle by its sides, determine if any of the sides are the same length. If no sides are the same length, then it is a **scalene triangle**. If two sides are the same length, then it is an **isosceles triangle**. If all three sides are the same length, then it is an **equilateral triangle**. You can show that two sides are the same length by drawing tick marks through the middle of the sides. Sides with a corresponding number of tick marks are the same length.



Scalene | Isosceles | Equilateral

Is an equilateral triangle an isosceles triangle?

Yes — an equilateral triangle **is** a special case of an isosceles triangle.

Here's the reasoning:

Definitions

- **Isosceles triangle**: A triangle with **at least two equal sides**.
- **Equilateral triangle**: A triangle with **all three sides equal**.

Logical Relationship

- Since an equilateral triangle has three equal sides, it automatically satisfies the condition of having at least two equal sides.
- Therefore, every equilateral triangle is isosceles.
- But not every isosceles triangle is equilateral (because most isosceles triangles only have two equal sides, not three).

Hierarchy

You can think of it like this:

Triangle Type	Condition	Examples
Equilateral	3 equal sides	All sides = 5 cm
Isosceles	≥ 2 equal sides	Sides = 5 cm, 5 cm, 8 cm
Scalene	No equal sides	Sides = 4 cm, 5 cm, 6 cm

So, equilateral triangles are a **subset** of isosceles triangles.

Does anyone or any textbook say that an isosceles triangle has exactly two sides, thus excluding equilateral triangles?

Yes — some sources do define an isosceles triangle as having *exactly* two equal sides, which excludes equilateral triangles. But many modern textbooks and references use the broader definition of "at least two equal sides," which includes equilateral triangles as a special case.

What different sources say

- **Wikipedia** notes both conventions:
  "Sometimes it is specified as having exactly two sides of equal length, and sometimes as having at least two sides of equal length, the latter version thus including the equilateral triangle as a special case." [1]
- **Math.net** and many school-level resources define it as *"at least two sides of equal length"*, which means equilateral triangles qualify. [2]
- **Other textbooks (especially older or elementary ones)** often prefer the "exactly two equal sides" definition, because it keeps the categories of equilateral and isosceles triangles distinct and non-overlapping for classification purposes.

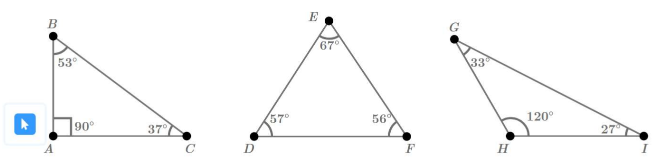
Why the difference?

- **Inclusive definition ("at least two equal sides")**:
  - Matches the formal mathematical logic.
  - Equilateral triangles are a subset of isosceles triangles.
  - Useful in proofs (e.g., the isosceles triangle theorem applies to equilateral triangles too).
- **Exclusive definition ("exactly two equal sides")**:
  - Simpler for teaching young students.
  - Keeps categories mutually exclusive: scalene, isosceles, equilateral.
  - Avoids confusion when first learning triangle types.

Practical takeaway

- In **higher-level mathematics**, the inclusive definition is more common, because it aligns with set theory and logical precision.
- In **elementary or middle school textbooks**, you'll often see the exclusive definition, because it's easier for classification exercises.

So, if you're teaching or writing, it's best to **clarify which definition you're using**. For your Math 111 students, the inclusive definition ("at least two equal sides") is more standard and mathematically rigorous, but you may want to note the alternative so students aren't confused if they encounter it elsewhere.



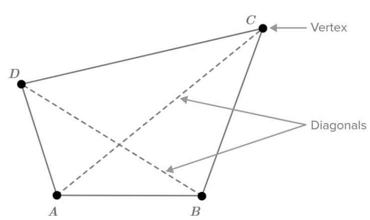
Right Acute Obtuse

Omit exterior angles

1.5

What is a Quadrilateral?

The prefix "quad-" means "four", and "lateral" is derived from the Latin word for "side". So a **quadrilateral** is a four-sided polygon. Since it is a polygon, you know that it is a two-dimensional figure made up of straight sides. A quadrilateral also has four angles formed by its four sides.



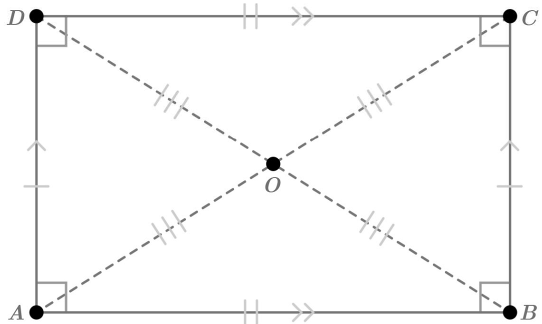
AB, BC, CD and DA are the sides and A, B, C and D are the vertices of the quadrilaterals.

Line segments AC and BD joining two non-consecutive vertices are called diagonals.

Two sides like AB **and** AD having a common endpoint are called adjacent sides.

Memorize

A **rectangle** is a quadrilateral with **four right angles**. All rectangles are parallelograms.



1. Opposite sides are parallel, i.e., $AB \parallel CD$ and $BC \parallel DA$.

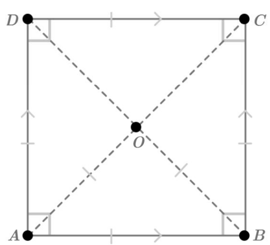
2. Opposite sides are congruent, i.e., $AB = CD$ and $BC = DA$.

3. All four of the angles are congruent and measure 90°, i.e., $\angle ABC = \angle BCD = \angle CDA = \angle DAB = 90^\circ$.

Memorize

Square and its Properties

A **square** is a quadrilateral with **four right angles** and **four congruent sides**. All squares are rectangles and rhombuses.

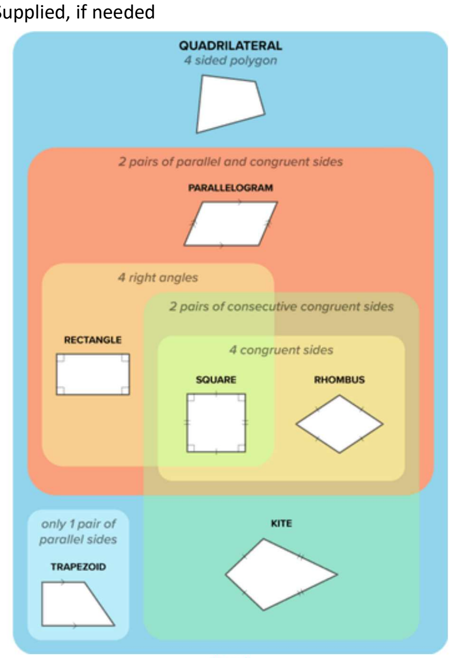


The Properties of a Square:

1. All four sides are congruent, i.e., $AB = BC = CD = DA$.

2. All the four angles are congruent and measures 90°, i.e., $\angle ABC = \angle BCD = \angle CDA = \angle DAB = 90^\circ$.

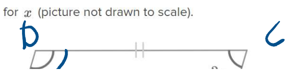
Supplied, if needed



[Figure 1]

Solving for Unknown Values of a Quadrilateral

Solve for x (picture not drawn to scale).



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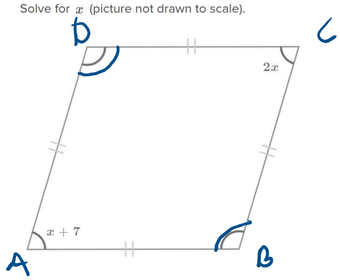
Solve for x (picture not drawn to scale).



$\angle A = \angle C$

$\angle B = \angle D$

This quadrilateral is marked as having **four congruent sides**, so it is a rhombus. Rhombuses have the same properties as parallelograms. One property of parallelograms is that **opposite angles are congruent**. This means that the marked angles in this rhombus must be congruent.

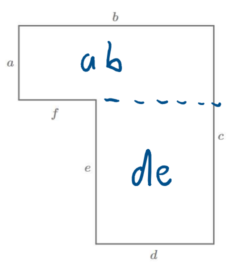
$$x + 7 = 2x$$
$$x = 7$$

1.6

Memorize

Perimeter

Perimeter is the distance around a shape. In other words, the total boundary length of a closed two-dimensional figure is called its perimeter. To find the perimeter of any two dimensional shape, find **the sum of the lengths** of all the sides.



Let P = perimeter

$p = a + b + c + d + e + f$

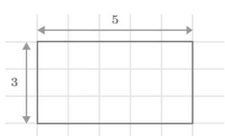
Let A = area

$A = ab + de$

Memorize

Area

Area is the amount of surface enclosed by a closed two-dimensional figure. It is measured by the number of **unit square**s it takes to cover a two-dimensional shape. For example, if you count the small squares, you will find there are 15 of them. Therefore, the **area** is $3 \cdot 5$ or 15 unit^2.

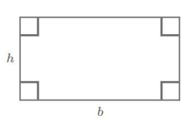


Memorize

Area of Rectangle

A rectangle is a very basic shape for area calculation. The **area** of a rectangle is **base** times **height**.

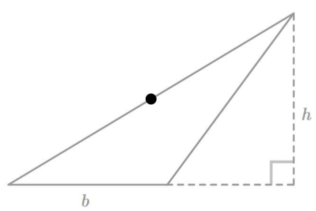
$$\text{Area}_{\text{rectangle}} = bh$$



Memorize

Area of Triangle

You can think of any triangle as **half a parallelogram**. If you rotate a triangle 180° about the midpoint of one of its sides, the original triangle and the new triangle will be a parallelogram.



Therefore, the **area** of a triangle is **base** times **height** divided by two.

Remember that any of the three sides can be the base. Also remember that the height must be perpendicular to the base and extend to the highest point of the triangle.

$$\text{Area}_{\text{triangle}} = \frac{bh}{2} = \frac{1}{2}bh$$

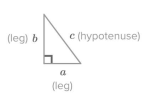
Memorize

Summary

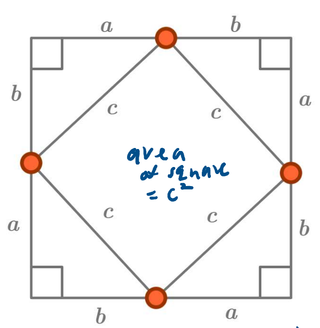
- **Perimeter** is the distance around a shape, found by summing the lengths of all sides.
- **Area** is the amount of surface enclosed by a closed two-dimensional figure.
- The area of a rectangle or parallelogram is calculated: $A = bh$
- The area of a triangle is calculated: $A = \frac{bh}{2}$

1.7
Memorize

The Pythagoras Theorem states that for right triangles with legs of lengths a and b and hypotenuse of length c, $a^2 + b^2 = c^2$.



The converse of the above is if we have a triangle with 3 sides: a, b, and c, and $a^2 + b^2 = c^2$, then the triangle is a right triangle.



area square of large $= (a+b)^2$

$= a^2 + 2ab + b^2$

$(a+b)(a+b)$

$a^2 + ab + ab + b^2$

$a^2 + 2ab + b^2$

area of each triangle

$= (\frac{1}{2})(ab)$

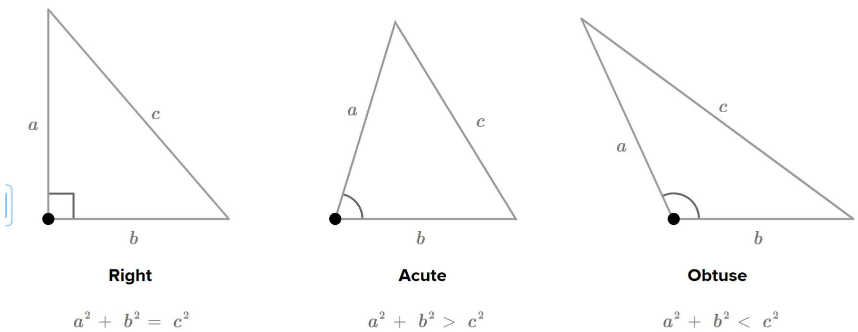
Area of 4 triangles $= (4)(\frac{1}{2})ab$

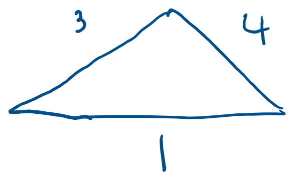
$= 2ab$

$a^2 + 2\cancel{ab} + b^2 = 2\cancel{ab} + c^2$

$$\boxed{a^2 + b^2 = c^2}$$

Supplied



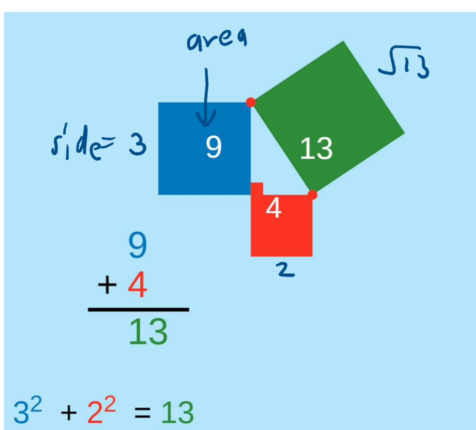


not to scale

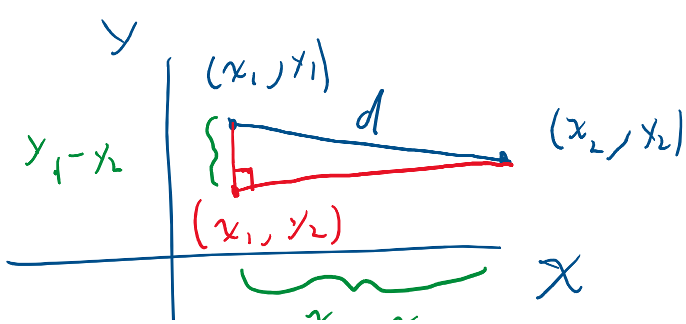
classify the triangle

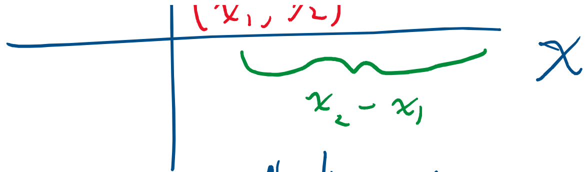
$1^2 = 1$
$3^2 = 9$
$4^2 = 16$

$1 + 9 < 16$
$1^2 + 3^2 < 4^2$
Let $a = 1$, $b = 3$, $c = 4$
$\therefore$ obtuse



Use the Pythagorean Theorem to derive the distance formula.





Let d = distance between points (x_1, y_1) and (x_2, y_2)

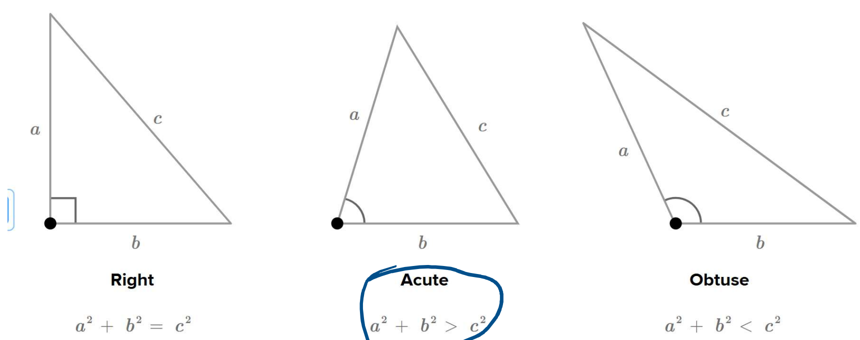
$(x_2-x_1)^2 + (y_1-y_2)^2 = d^2$

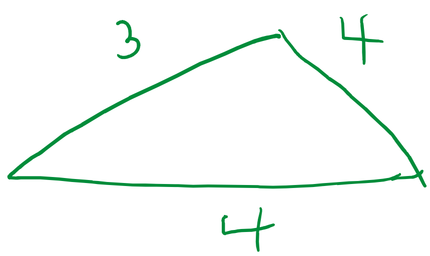
$(x_2-x_1)^2 + (y_2-y_1)^2 = d^2$

$\boxed{d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}}$ supplied

Your Name MTH 111 quiz 6 write each problem.

1.





Classify the triangle using the above. Can you classify it in another way?

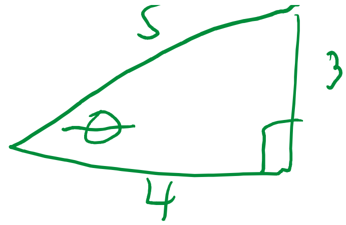
isosceles
2 equal sides

$3^2 = 9$
$4^2 = 16$
$4^2 = 16$

$9 + 16 > 16$
acute

2.

Find $\tan(\theta)$



Find tan(θ)

$$\tan\theta = \frac{opp}{adj} = \boxed{\frac{3}{4}}$$

3. Solve $8x - 4 = 2x + 3$ and show your check.

$$6x = 7$$

$$\boxed{x = \frac{7}{6}}$$

check

$$8\left(\frac{7}{6}\right) - 4 \stackrel{?}{=} 2\left(\frac{7}{6}\right) + 3$$

$$\frac{4(7)}{3} - 4 \stackrel{?}{=} \frac{7}{3} + 3$$

$$\frac{28}{3} - \frac{4(3)}{(3)} \stackrel{?}{=} \frac{7}{3} + \frac{3(3)}{3}$$

$$\frac{28-12}{3} \stackrel{?}{=} \frac{7+9}{3}$$

$$\frac{16}{3} = \frac{16}{3} \checkmark$$

4. Convert 200 millimeters to meters. Carry units throughout the calculation.

$$200\ mm = (200\ \cancel{mm})\left(\frac{1\ m}{1000\ \cancel{mm}}\right)$$

$$= \frac{2\cancel{0}\cancel{0}}{10\cancel{0}\cancel{0}}\ m = \left(\frac{2}{10}\right) m = \boxed{\left(\frac{1}{5}\right) m = 0.2\ m}$$

5. Find the x-intercept and y-intercept of the line given by $6x - 4y = 12$. Use these to graph the line.

x-intercept	y-intercept
$6x - (4)(0) = 12$	$(6)(0) - 4y = 12$

x-intercept

$6x - (4)(0) = 12$

$6x = 12$

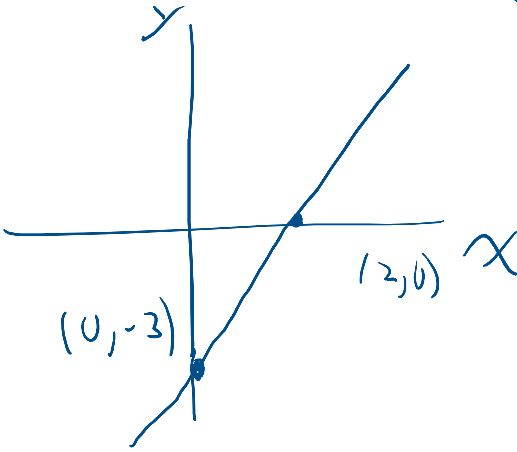
$\boxed{x = 2 \text{ or } (2,0)}$

y-intercept

$(6)(0) - 4y = 12$

$-4y = 12$

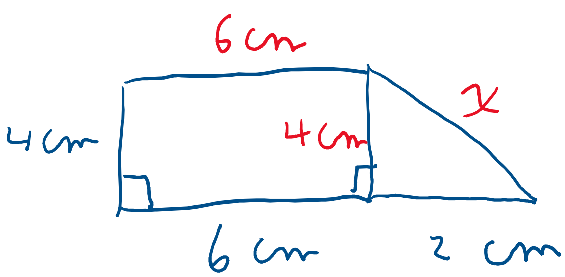
$\boxed{y = -3 \text{ or } (0,-3)}$



6. Write 2581 in scientific notation.

$\boxed{2.581 \times 10^3}$

7.



Find the perimeter and area of the figure.

Area = area of rectangle + area of triangle

Area = area of rectangle + area of triangle

$$= (4\text{ cm})(6\text{ cm}) + \left(\tfrac{1}{2}\right)(2\text{ cm})(4\text{ cm})$$

$$= 24\text{ cm}^2 + 4\text{ cm}^2$$

$$= \boxed{28\text{ cm}^2}$$

$$\text{Perimeter} = 4\text{ cm} + 6\text{ cm} + 6\text{ cm} + 2\text{ cm} + x$$

$$= 18\text{ cm} + x$$

$$x = \sqrt{(4\text{ cm})^2 + (2\text{ cm})^2}$$

$$= \sqrt{(16+4)\text{ cm}^2}$$

$$= \sqrt{20}\text{ cm} \approx 4.5\text{ cm}$$

Sqrt(20)=4.47213595499958

$$\text{Perimeter} \approx (18 + 4.5\ \text{cm}$$

$$= \boxed{22.5\text{ cm}}$$