

3.4 Understand Slope of a Line-optional

3.4 Exercise Set, page 409 (391): 1, 3, 9, 10, 13, 19, 28

3.5 Use the Slope-Intercept Form of an Equation of a Line-optional

3.5 Exercise Set, page 451 (433): 1, 4, 7, 9, 25, 29, 37, 42, 44

Exam 2, Friday, 10/24/25, 2.1 - 2.4, 3.1 - 3.5

5. Trigonometry

5.1 Use Properties of Angles, Triangles, and the Pythagorean and Theorem

5.1 Exercise Set, page 612 (594): 1, 5, 7, 9, 13, 15, 22

5.2 Applications: Sine, Cosine and Tangent Ratios

5.2 Exercise Set, page 640 (622): 1, 5, 7, 11, 15, 16, 19, 26

I will supply supplementary material about converting between decimal degrees and DMS notation.

after class notes

Is $x = 4$ a solution

$$\text{of } 2x - \frac{1}{2} = x + 3 \text{ ?}$$

$$2(4) - \frac{1}{2} \stackrel{?}{=} 4 + 3$$

$$8 - \frac{1}{2} \stackrel{?}{=} 7$$

$$\frac{8}{1} \neq 7 + \frac{1}{2}$$

$$8 \stackrel{?}{=} \frac{7(2)}{2} + \frac{1}{2}$$

$$\frac{8(2)}{2} \stackrel{?}{=} \frac{14 + 1}{2}$$

$$\frac{16}{2} \neq \frac{15}{2}$$

$\therefore x = 4$ is not a solution

$$\text{Is } (x, y) = (6, -3)$$

a solution of $8x + 16y = 0$?

$$8(6) + 16(-3) \stackrel{?}{=} 0$$

$$48 - 48 \stackrel{?}{=} 0$$

$$0 = 0 \checkmark$$

$\therefore (x, y) = (6, -3)$ is a solution.

16. Solve the formula $A = \frac{1}{2}h(b_1 + b_2)$ for b_1 .

$$\frac{2A}{h} = 2 \left(\frac{1}{2} \right) \frac{h}{h} (b_1 + b_2)$$

$$\frac{2A}{h} = b_1 + b_2$$

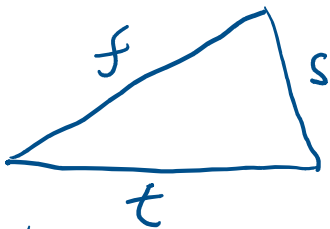
$$b_1 + b_2 = \frac{2A}{h}$$

$$b_1 = \frac{2A}{h} - b_2$$

$$b_1 = \frac{2A - b_2 h}{h}$$

In the following exercises, solve using a geometry formula.

26. The perimeter of a triangle is 39 feet. One side of the triangle is one foot longer than the second side. The third side is two feet longer than the second side. Find the length of each side.



Let P = perimeter of triangle
 $P = 39 \text{ ft}$

Let f = length of first side

$s = 11$ // 2nd side

$t = 11$ // 3rd side

$$P = f + s + t = 39 \text{ ft}$$

$$f = s + 1 \text{ ft}$$

$$t = s + 2 \text{ ft}$$

$$f + s + t = (s + 1 \text{ ft}) + s + (s + 2 \text{ ft}) = 39 \text{ ft}$$

$$\Rightarrow 3s + 3 \text{ ft} = 39 \text{ ft}$$

solve for s

$$3s = 39 \text{ ft} - 3 \text{ ft}$$

$$3s = 39 \text{ ft} - 3 \text{ ft}$$

$$3s = 36 \text{ ft}$$

$$s = \frac{36 \text{ ft}}{3}$$

$$s = 12 \text{ ft}$$

$$f = 12 \text{ ft} + 1 \text{ ft}$$

$$f = 13 \text{ ft}$$

$$t = 12 \text{ ft} + 2 \text{ ft}$$

$$t = 14 \text{ ft}$$

The lengths of the 3 sides of the triangle are 12 feet, 13 feet, and 14 feet.

Check

$$13 = 12 + 1$$

$$14 = 12 + 2$$

$$12 + 13 + 14 = 39$$

memorize

$$\text{Area of triangle } A = \frac{bh}{2}$$

Where A = area, b = base, h = height

5.1

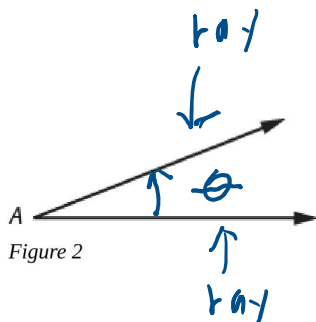


Figure 2

$\theta = \text{theta}$

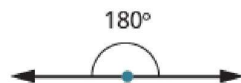


Figure 1

Straight angle

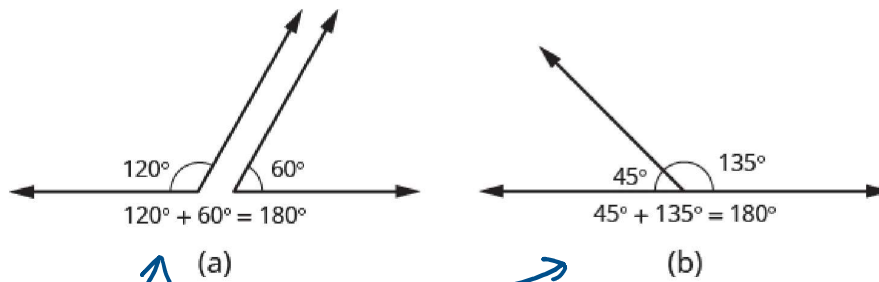


Figure 3

Memorize

Supplementary and Complementary Angles

If the sum of the measures of two angles is 180° , then the angles are supplementary.

If $\angle A$ and $\angle B$ are supplementary, then $m\angle A + m\angle B = 180^\circ$.

If the sum of the measures of two angles is 90° , then the angles are complementary.

If $\angle A$ and $\angle B$ are complementary, then $m\angle A + m\angle B = 90^\circ$.

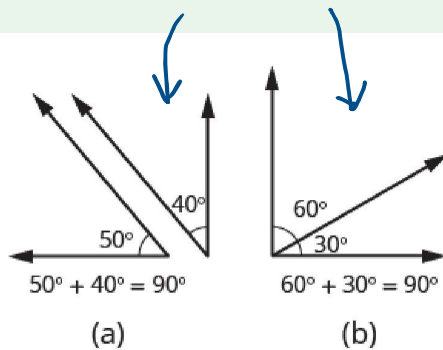
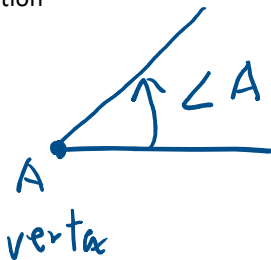


Figure 4

notation



$$m\angle A = \angle A$$

↑
measure of angle A

Memorize

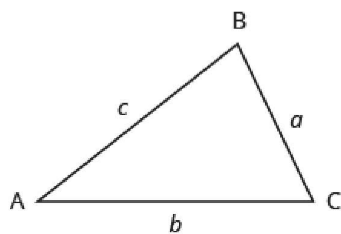


Figure 5

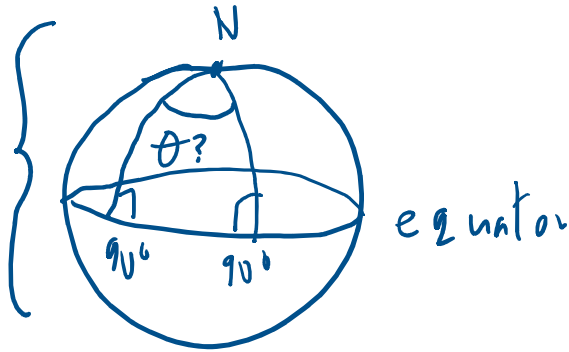
Sum of the Measures of the Angles of a Triangle

For any $\triangle ABC$, the sum of the measures of the angles is 180° .

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

triangle in the plane

not
on
exam



$$90^\circ + 90^\circ + \theta \geq 180^\circ$$

Memorize: a right triangle is a triangle containing a right angle.

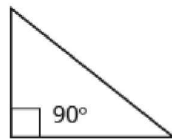
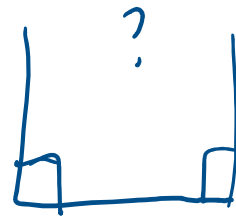


Figure 6

Could a right triangle have two right angles?

No

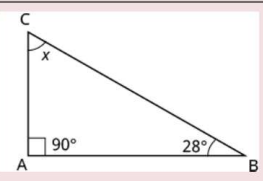


0°
90° 90° squashed Δ ?

EXAMPLE 4

One angle of a right triangle measures 28° . What is the measure of the third angle?

Solution

Step 1. Read the problem. Draw the figure and label it with the given information.	
Step 2. Identify what you are looking for.	the measure of an angle
Step 3. Name . Choose a variable to represent it.	let x = the measure of the angle
Step 4. Translate . Write the appropriate formula and substitute.	$m\angle A + m\angle B + m\angle C = 180$
Step 5. Solve the equation.	$x + 90 + 28 = 180$ $x + 118 = 180$ $x = 62$
Step 6. Check : $180 \stackrel{?}{=} 90 + 28 + 62$ $180 = 180 \checkmark$	
Step 7. Answer the question.	The measure of the third angle is 62° .

$$x + 28^\circ = 90^\circ$$

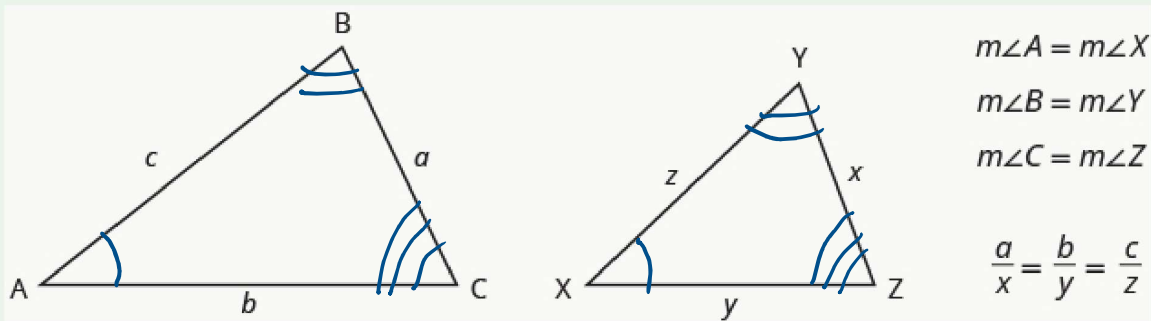
$$x = 90^\circ - 28^\circ$$

$$x = 62^\circ$$

Memorize

Properties of Similar Triangles

If two triangles are similar, then their corresponding angle measures are equal and their corresponding side lengths are in the same ratio.



$$A \text{ is } \frac{a}{b} = \frac{x}{y}$$

$$a = bc$$

$$b = Ac$$

$$c = Ab$$

$$\frac{a}{x} = \frac{b}{y}$$

$$\left(\frac{a}{x}\right)x = \left(\frac{b}{y}\right)x$$

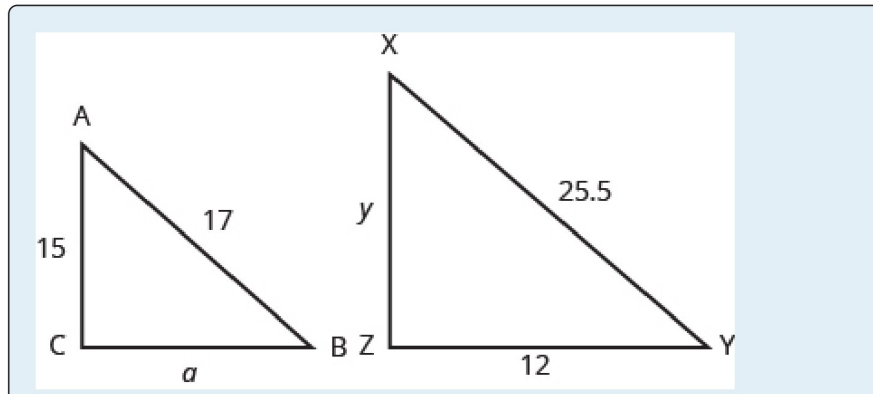
$$a = \frac{bx}{y}$$

$$a = bx$$

$$c = \sqrt{a^2 + b^2}$$

$$\frac{a}{b} = \frac{bx}{by}$$

$$\frac{a}{b} = \frac{x}{y}$$



$\triangle ABC$ is similar to $\triangle XYZ$

Find a

$$\frac{a}{12} = \frac{17}{25.5}$$

$$a = \frac{(17)(12)}{25.5}$$

$$17 \cdot 12 / 25.5 = 8$$

$$a^2 + 15^2 = 17^2$$

$$a^2 = 17^2 - 15^2$$

$$a = \sqrt{17^2 - 15^2}$$

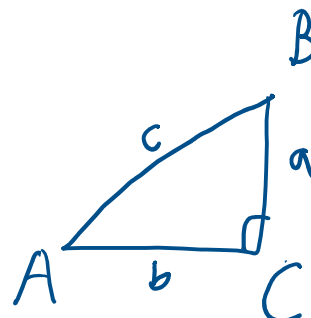
$$\text{Sqrt}(17^2 - 15^2) = 8$$

Memorize

The Pythagorean Theorem

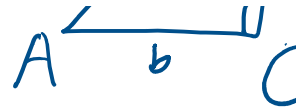
In any right triangle $\triangle ABC$,

$$a^2 + b^2 = c^2$$



in any right triangle $\triangle ABC$,

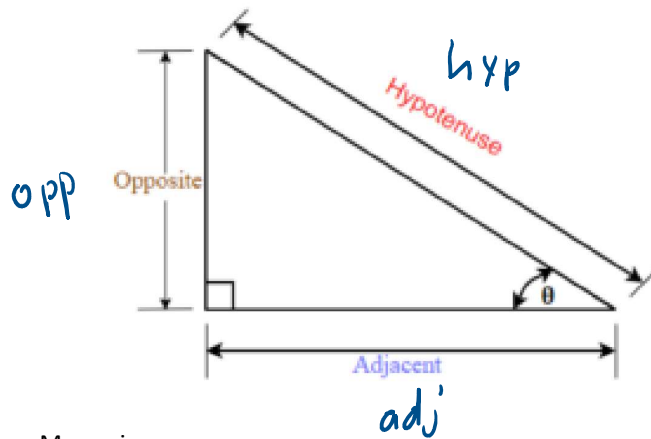
$$a^2 + b^2 = c^2$$



Converse: if in a triangle, with sides a , b , and c , we have $a^2 + b^2 = c^2$,
Then the triangle is a right triangle.

5.2

Memorize



Memorize

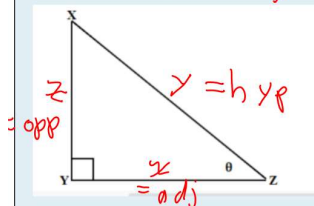
Three Basic Trigonometric Ratios

- $\sin \theta = \frac{\text{the length of the opposite side}}{\text{the length of the hypotenuse side}}$ $= \sin \theta = \frac{\text{opp}}{\text{hyp}}$
- $\cos \theta = \frac{\text{the length of the adjacent side}}{\text{the length of the hypotenuse side}}$ $= \cos \theta = \frac{\text{adj}}{\text{hyp}}$
- $\tan \theta = \frac{\text{the length of the opposite side}}{\text{the length of the adjacent side}}$ $= \tan \theta = \frac{\text{opp}}{\text{adj}}$

Where θ is the measure of a reference angle measured in degrees.

TRY IT 2

For the given triangle find the sine, cosine, and tangent ratio.



Show answer

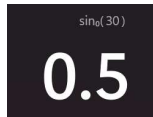
$$\sin \theta = \frac{z}{y}$$

$$\cos \theta = \frac{x}{y}$$

$$\tan \theta = \frac{z}{x}$$

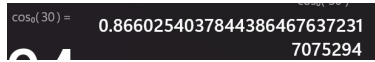
$$\theta = \angle Z$$

Windows scientific calculator

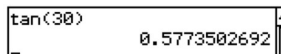


$$\sin(30^\circ) = 0.5$$

$$\cos(30^\circ) \approx 0.87$$

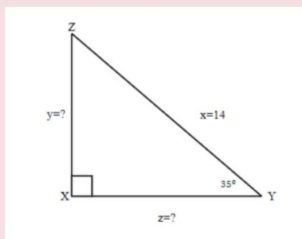


$$\tan(30^\circ) \approx 0.58$$



EXAMPLE 5

Find the missing sides. Round your final answer to two decimal places



$$\sin 35^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{y}{14} \Rightarrow y = 14 \sin 35^\circ \approx 8.02$$

$$\cos 35^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{z}{14} \Rightarrow z = 14 \cos 35^\circ \approx 11.47$$

$$\begin{array}{l} 14 * \sin(35) \\ 14 * \cos(35) \\ \hline \end{array} \begin{array}{l} 8.030070109 \\ 11.46812862 \end{array}$$

Review questions

1. What is a solution of an equation? Answer in one sentence.

A solution of an equation is a value of each variable that makes the equation true.

A solution of an equation is a value that when substituted for a variable satisfies the equation.

2. Solve and show check. $7x + \frac{1}{3} = 3x - \frac{2}{9}$

$$\begin{aligned} 7x - 3x &= -\frac{2}{9} - \frac{1}{3} \\ 4x &= -\frac{2}{9} - \frac{1}{3} \left(\frac{3}{3} \right) \\ 4x &= -\frac{2}{9} - \frac{3}{9} \\ 4x &= -\frac{5}{9} \end{aligned}$$

$$4x = -\frac{5}{9}$$

$$x = -\frac{5}{36}$$

$$7\left(-\frac{5}{36}\right) + \frac{1}{3} \stackrel{?}{=} 3\left(-\frac{5}{36}\right) - \frac{2}{9}$$

$$-\frac{35}{36} + \frac{12}{36} \stackrel{?}{=} -\frac{15}{36} - \frac{8}{36}$$

$$-\frac{23}{36} = -\frac{23}{36} \checkmark$$

$$7x + \frac{1}{3} = 3x - \frac{2}{9}$$

$$9(7x) + 9\left(\frac{1}{3}\right) = 9(3x) - 9\left(\frac{2}{9}\right)$$

$$63x + 3 = 27x - 2$$

$$(63 - 27)x = -5$$

$$x = -\frac{5}{36}$$

Find the slope of the line given by $-x + 3y = 5$.

$$3y = x + 5$$

$$y = \left(\frac{1}{3}\right)x + \frac{5}{3}$$

slope-intercept
equation
of the line

$$\text{slope} = \frac{1}{3}$$

Find x -intercept and y -intercept

Then calculate slope = $\frac{y_2 - y_1}{x_2 - x_1}$

x -intercept

$$-x + (3)(0) = 5$$

$$-x = 5$$

$$x = -5$$

$$(-5, 0)$$

y -intercept

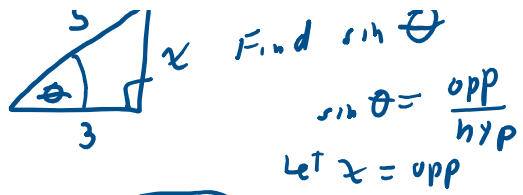
$$-0 + 3y = 5$$

$$y = \frac{5}{3}$$

$$(0, \frac{5}{3})$$

$$\text{slope} = \frac{\frac{5}{3} - 0}{0 - (-5)} = \frac{\frac{5}{3}}{5} = \frac{5}{3} \cdot \frac{1}{5} = \frac{1}{3}$$





$$x = \sqrt{5^2 - 3^2}$$

$$x = \sqrt{25 - 9}$$

$$x = \sqrt{16}$$

$$x = 4$$

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \boxed{\frac{4}{5}}$$