10-10-25 MTH 111

3. Equations and their Graphs

3.1 Use the Rectangular Coordinate System
3.1 Exercise Set, page 312 (294): 1, 5, 7, 11, 15, 19, 25
3.2 Graph Linear Equations in Two Variables
3.2 Exercise Set, page 346 (328): 1, 3, 21, 25, 27, 33
3.3 Graphs with Intercepts-optional
3.3 Exercise Set, page 373 (355): 10,16

3.4 Understand Slope of a Line-optional
3.4 Exercise Set, page 409 (391): 1, 3, 9, 10, 13, 19, 28
3.5 Use the Slope-Intercept Form of an Equation of a Line-optional
3.5 Exercise Set, page 451 (433): 1, 4, 7, 9, 25, 29, 37, 42, 44

Exam 2, Friday, 10/24/25, 2.1 - 2.4, 3.1 - 3.5

After class notes

$7x - 4y = 6$

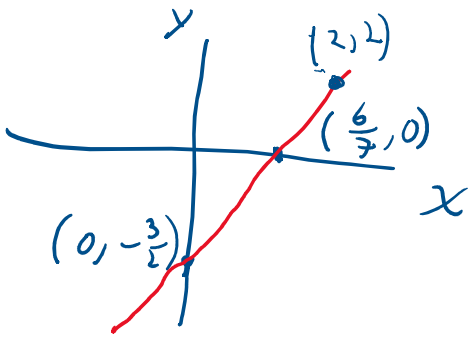
Find the x-intercept and y-intercept
Use them to graph the line

x-intercept
set $y=0$, solve for x

$7x - (4)(0) = 6$
$7x - 0 = 6$
$7x = 6$
$\boxed{x = \frac{6}{7}}$

y-intercept
set $x=0$, solve for y

$(7)(0) - 4y = 6$
$0 - 4y = 6$
$-4y = 6$
$\boxed{y = \frac{6}{-4} = -\frac{3}{2}}$



Find a 3rd point
Let $x=2$

$\Rightarrow (7)(2) - 4y = 6$
$14 - 4y = 6$
$14 - 14 - 4y = 6 - 14$
$(14-14) - 4y = -8$
$0 - 4y = -8$
$-4y = -8$
$\boxed{y = 2}$

Graph the line by finding and plotting the intercepts and a third point

$10x - 5y = 3$

x-intercept

$10x - (5)(0) = 3$

$10x = 3$

$x = \frac{3}{10}$

y-intercept

$(10)(0) - 5y = 3$

$-5y = 3$

$y = -\frac{3}{5}$

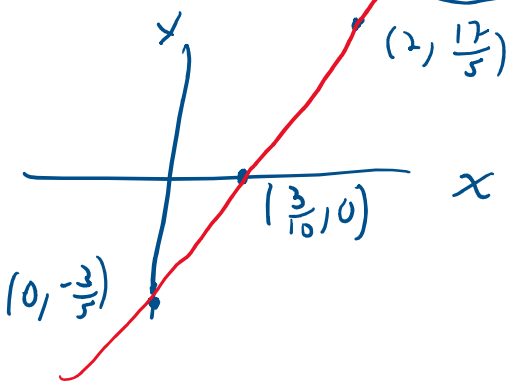
3rd point Let $x = 2$, solve for y

$(10)(2) - 5y = 3$

$20 - 5y = 3$

$-5y = -17$

$y = \frac{-17}{-5} = \frac{17}{5}$



Notation

$-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$

all are equal but we usually put the minus sign in front of the fraction or in the numerator

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3.4

Memorize

Slope of a line

The slope of a line of a line is $m = \dfrac{\text{rise}}{\text{run}}$.

The rise measures the vertical change and the run measures the horizontal change between two points on the line.

$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Δ = change

Memorize

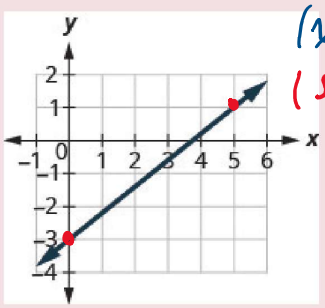
Positive and negative slopes

We 'read' a line from left to right just like we read words in English. As you read from left to right, the line is going up; it has positive slope. The line is going down; it has negative slope.

EXAMPLE 1

How to Use $m = \dfrac{\text{rise}}{\text{run}}$ to Find the Slope of a Line from its Graph

Find the slope of the line shown.



(x_2, y_2)
$(5, 1)$
(x_1, y_1)
$(0, -3)$

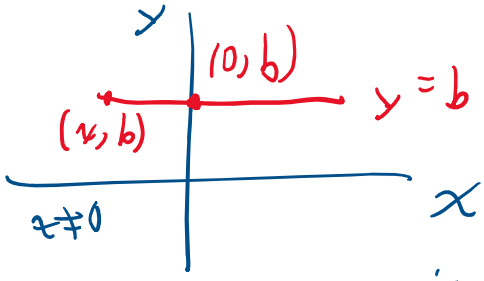
Solution

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-3)}{5-0} = \frac{1+3}{5} = \boxed{\frac{4}{5}}$$

$$m = \frac{-3-1}{0-5} = \frac{-4}{-5} = \boxed{\frac{4}{5}}$$

Memorize

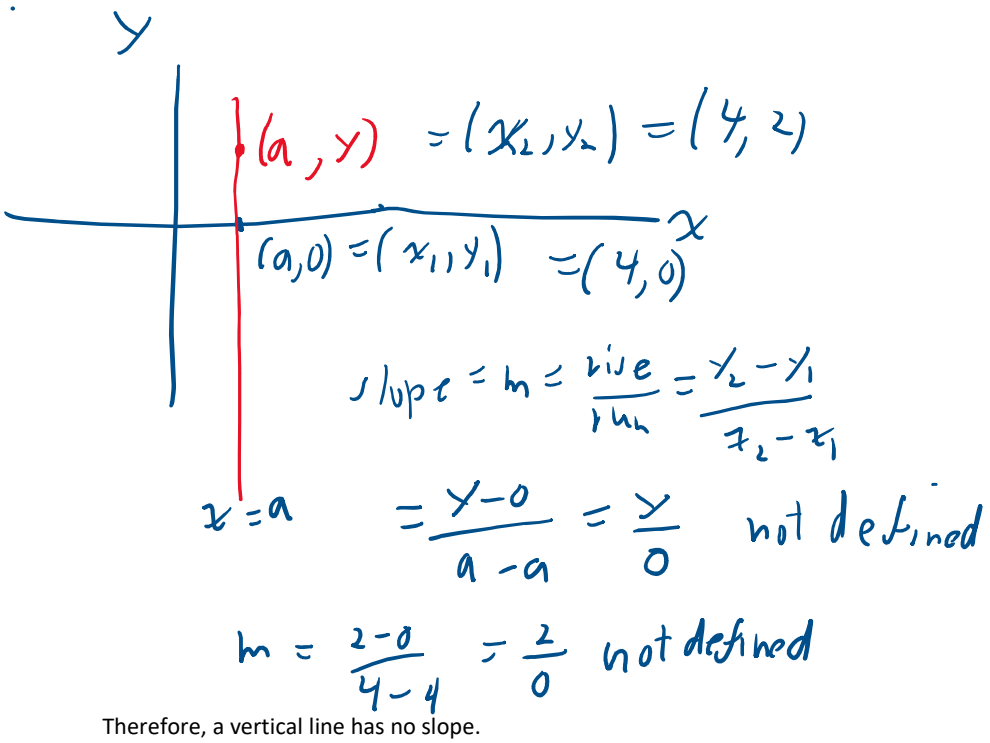
Horizontal line $y = b$ **Vertical line** $x = a$



x = variable
b = constant
y = variable

$$\text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

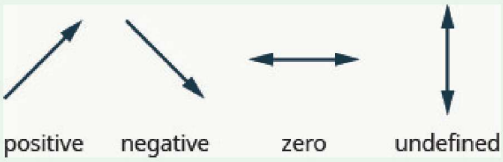
$$= \frac{b-b}{0-x} = \frac{0}{-x} = 0$$



Therefore, a vertical line has no slope.

Memorize

Quick guide to the slopes of lines



Remember, we 'read' a line from left to right, just like we read written words in English.

Memorize

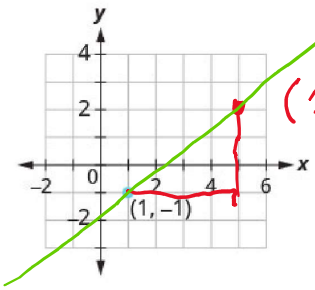
(x_1, y_1) read ' x sub 1, y sub 1'
(x_2, y_2) read ' x sub 2, y sub 2'

EXAMPLE 7

How To Graph a Line Given a Point and The Slope

Graph the line passing through the point $(1, -1)$ whose slope is $m = \frac{3}{4}$. = rise/run

Solution

Step 1. Plot the given point.	Plot (1, –1).	

(x2, y2)

$x_2 = 1 + 4 = 5$
$y_2 = -1 + 3 = 2$
$\Rightarrow (x_2, y_2) = (5, 2)$

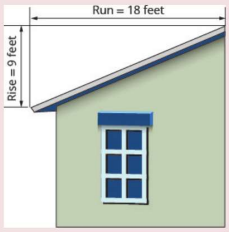
Memorize

Graph a line given a point and the slope.

1. Plot the given point.
2. Use the slope formula $m = \dfrac{\text{rise}}{\text{run}}$ to identify the rise and the run.
3. Starting at the given point, count out the rise and run to mark the second point.
4. Connect the points with a line.

EXAMPLE 10

The 'pitch' of a building's roof is the slope of the roof. Knowing the pitch is important in climates where there is heavy snowfall. If the roof is too flat, the weight of the snow may cause it to collapse. What is the slope of the roof shown?



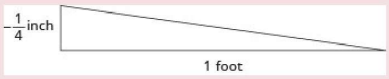
Solution

Use the slope formula.	$m = \dfrac{\text{rise}}{\text{run}}$
Substitute the values for rise and run.	$m = \dfrac{9}{18}$
Simplify.	$m = \dfrac{1}{2}$
The slope of the roof is $\dfrac{1}{2}$.	
	The roof rises 1 foot for every 2 feet of horizontal run.

insert units
here, the units cancel

EXAMPLE 11

Have you ever thought about the sewage pipes going from your house to the street? They must slope down $\dfrac{1}{4}$ inch per foot in order to drain properly. What is the required slope?



Solution

Use the slope formula.	$m = \dfrac{\text{rise}}{\text{run}}$ $m = \dfrac{-\frac{1}{4}\text{inch}}{1\text{ foot}}$ $m = \dfrac{-\frac{1}{4}\text{inch}}{12\text{ inches}}$
Simplify.	$m = -\dfrac{1}{48}$
	The slope of the pipe is $-\dfrac{1}{48}$.

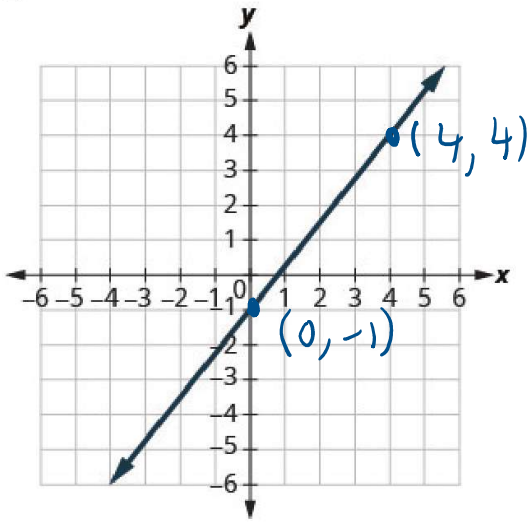
$= -\frac{1}{4}\frac{\text{in}}{\text{ft}}$

The pipe drops 1 inch for every 48 inches of horizontal run.

3.4

In the following exercises, find the slope of each line shown.

2.



$$\text{slope} = m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-1)}{4 - 0} = \frac{4+1}{4} = \frac{5}{4}$$

3.4

In the following exercises, find the slope of each line.

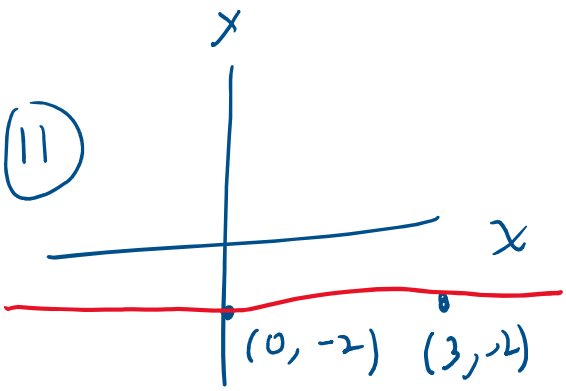
11. $y = -2$

12. $x = -5$

Graph each line.
Hint: plot 2 points on each line.

$$m = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

(11)

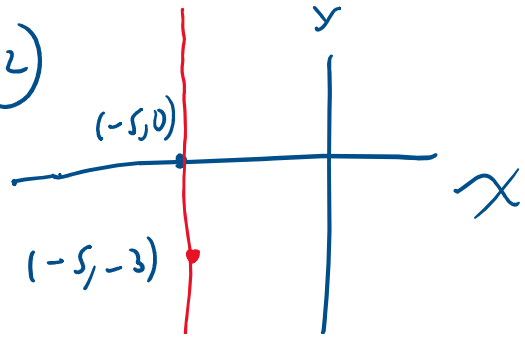


$y = -2$

$$m = \frac{-2-(-2)}{3-0} = \frac{-2+2}{3} = \frac{0}{3} = 0$$

slope = 0

(12)



(-5,-3)

$m = \frac{0-(-3)}{(-5)-(-5)} = \frac{3}{0}$ not defined

3.4

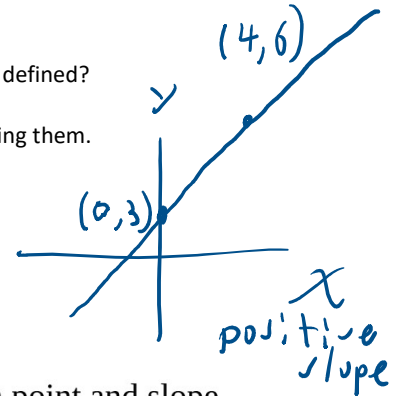
In the following exercises, use the slope formula to find the slope of the line between each pair of points.

14. $(0,3), (4,6)$

Is the slope positive, negative, zero, or not defined?

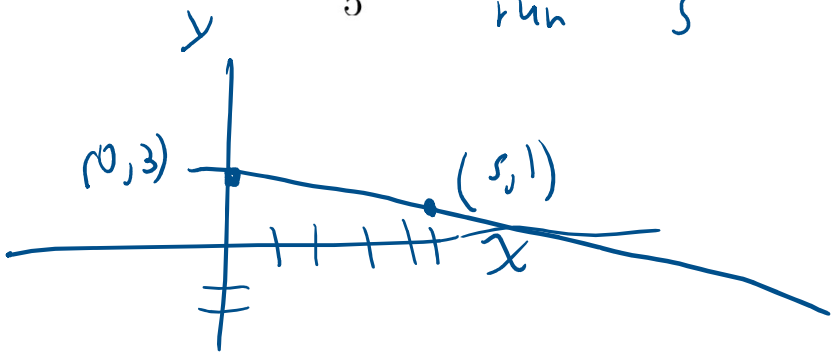
Plot the points and graph the line containing them.

$m = \frac{y_2-y_1}{x_2-x_1} = \frac{6-3}{4-0} = \frac{3}{4} > 0$



In the following exercises, graph each line with the given point and slope.

23. y-intercept 3; $m = -\frac{2}{5}$ (0,3) $= \frac{\text{rise}}{\text{run}} = \frac{-2}{5}$



$y_2 = y_1 - 2 = 3 - 2 = 1$

$x_2 = x_1 + 5 = 0 + 5$

3.4: 28

28. A local road has a grade of 6%. The grade of a road is its slope expressed as a percent. Find the slope of the road as a fraction and then simplify. What rise and run would reflect this slope or grade?

the slope of the road as a fraction and then simplify. What rise and run would reflect this slope or grade?

slope $= 6\% = 6\left(\frac{1}{100}\right) = \frac{6}{100} = \boxed{\frac{3}{50}}$

rise $= 3$

run $= 50$

3, 5 $y = mx + b$ is slope-intercept form of the equation of a line.

m = slope

Let $x = 0 \Rightarrow y = m(0) + b = 0 + b = b$

$\therefore b = y$-intercept

Textbook notation

$y = \frac{1}{2}x + 3$

$y = \left(\frac{1}{2}\right)x + 3$ or $y = \frac{x}{2} + 3$

bad notation

$y = 1/2\,x + 3$

not clear where does x go?

Memorize

Slope-intercept form of an equation of a line

The slope–intercept form of an equation of a line with slope m and y-intercept, $(0, b)$ is,
$y = mx + b$

Find the slope of the line given by $4x - 7y = 10$

and write the equation in slope-intercept form.

$-7y = -4x + 10$

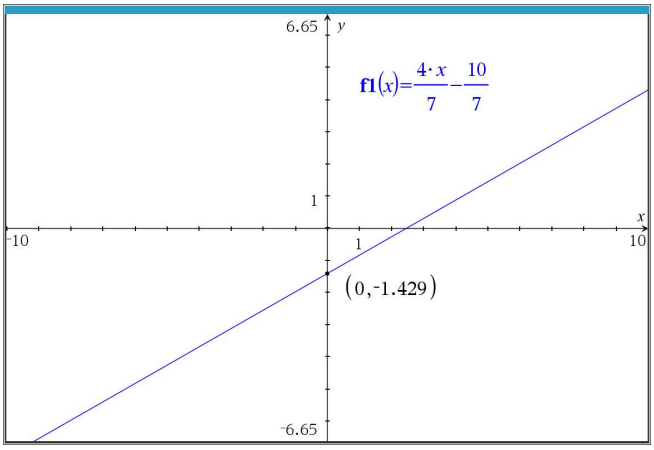
$y = \frac{-4x}{-7} + \frac{10}{-7}$

$4x \quad 10$

$y = \frac{4x}{7} - \frac{10}{7}$

$m = \frac{4}{7}$

-10/7=-1.4286



TI-nspire graphing calculator

memorize

Methods to graph lines

Methods to Graph Lines			
Point Plotting	**Slope–Intercept**	**Intercepts**	**Recognize Vertical and Horizontal Lines**
x | y	$y = mx + b$	x | y 0 | | 0	
Find three points. Plot the points, make sure they line up, then draw the line.	Find the slope and *y*-intercept. Start at the *y*-intercept, then count the slope to get a second point.	Find the intercepts and a third point. Plot the points, make sure they line up, then draw the line.	The equation has only one variable. $x = a$ vertical $y = b$ horizontal

Memorize

Parallel lines

Parallel lines are lines in the same plane that do not intersect.

- Parallel lines have the same slope and different *y*-intercepts.
- If m_1 and m_2 are the slopes of two parallel lines then $m_1 = m_2$.
- Parallel vertical lines have different *x*-intercepts.

Supplied

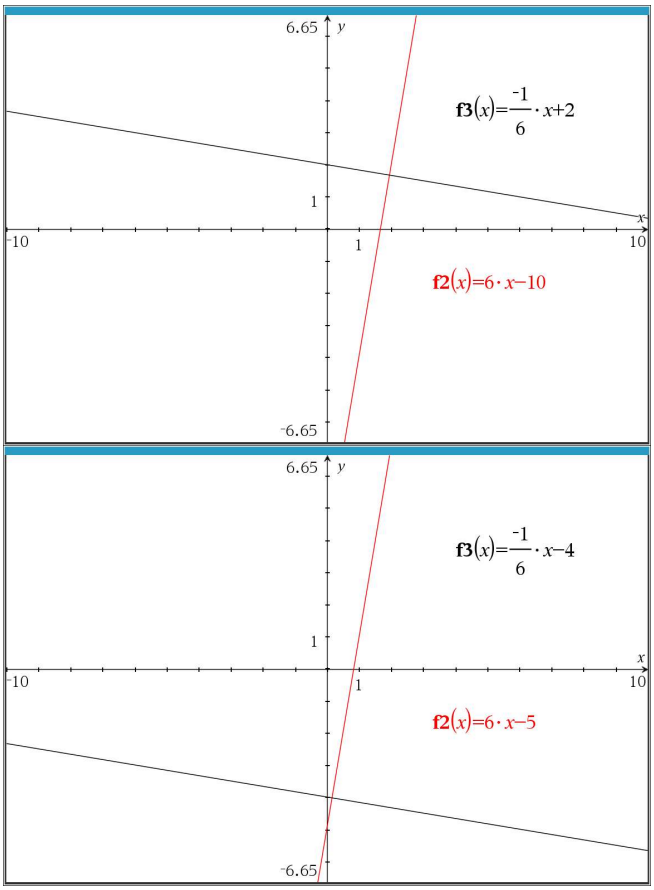
Perpendicular lines

Perpendicular lines are lines in the same plane that form a right angle.

If m_1 and m_2 are the slopes of two perpendicular lines, then:

$m_1 \cdot m_2 = -1$ and $m_1 = \dfrac{-1}{m_2}$

Vertical lines and horizontal lines are always perpendicular to each other.



Your Name MTH 111 bonus quiz 2 Write each problem

1. Solve and show your check $8x - 5 = \frac{x}{2}$

$(8x)\ 2 \cdot \frac{x}{2} = 5 \quad 8x - \frac{x}{2} = 5 \quad \rightarrow 15x = 10$

1. Solve and show your check $8x - 5 = \frac{x}{2}$

$8x - \frac{x}{2} = 5 \rightarrow 15x = 10$

$16x - x = 10$

$x = \frac{10}{15}$

$x = \frac{2}{3}$

$\frac{8x}{2} - \frac{x}{2} = 5$

$\frac{16x - x}{2} = 5$

$\frac{15x}{2} = 5$

$x = \frac{2(5)}{15} = \frac{2}{3}$

invert and multiply

$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c}$

check

$8\left(\frac{2}{3}\right) - \frac{\frac{2}{3}}{2} \stackrel{?}{=} 5$

$\frac{16}{3} - \frac{1}{3} \stackrel{?}{=} 5$

$\frac{15}{3} \stackrel{?}{=} 5 \Rightarrow 5 = 5$ ✓

$\frac{\frac{2}{3}}{2} = \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$

2. Solve $V = \frac{4\pi r^3}{3}$ for r.

$\frac{V(3)}{4\pi} = \left(\frac{3}{4\pi}\right)\left(\frac{4\pi}{3}\right) r^3$

$\frac{3V}{4\pi} = r^3$

$r^3 = \frac{3V}{4\pi}$

$\sqrt[3]{r^3} = \sqrt[3]{\frac{3V}{4\pi}}$

$r = \sqrt[3]{\frac{3V}{4\pi}}$

shorter presentation

$V = \frac{4\pi r^3}{3}$

$\frac{3V}{4\pi} = r^3 \Rightarrow r = \sqrt[3]{\frac{3V}{4\pi}}$

3. Find the slope of the line passing through the points $(7, 2)$ and $(10, 16)$.

$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{16 - 2}{10 - 7} = \frac{14}{3}$

4. Write $2x - 9y = 18$ in slope-intercept form.

$y = mx + b$

$-9y = -2x + 18$

$y = \frac{-2x}{-9} + \frac{18}{-9}$

$y = \frac{2x}{9} - 2 \Leftrightarrow y = \left(\frac{2}{9}\right)x - 2 \Rightarrow m = \frac{2}{9}, \; b = -2$

5. Is the point $(0, 2)$ on the line from #4? Why or why not?

$2(0) - 9(2) \stackrel{?}{=} 18$

$0 - 18 \stackrel{?}{=} 18$

$-18 \stackrel{?}{=} 18$

No

$\therefore (0, 2)$ is not on the line of #4

#4 $\Rightarrow$ y intercept is $(0, -2)$

$\therefore (0, 2)$ is not on the line