

2. Solving Linear Equations and Inequalities

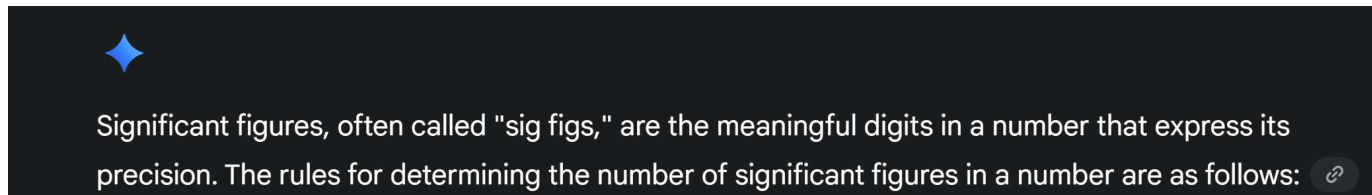
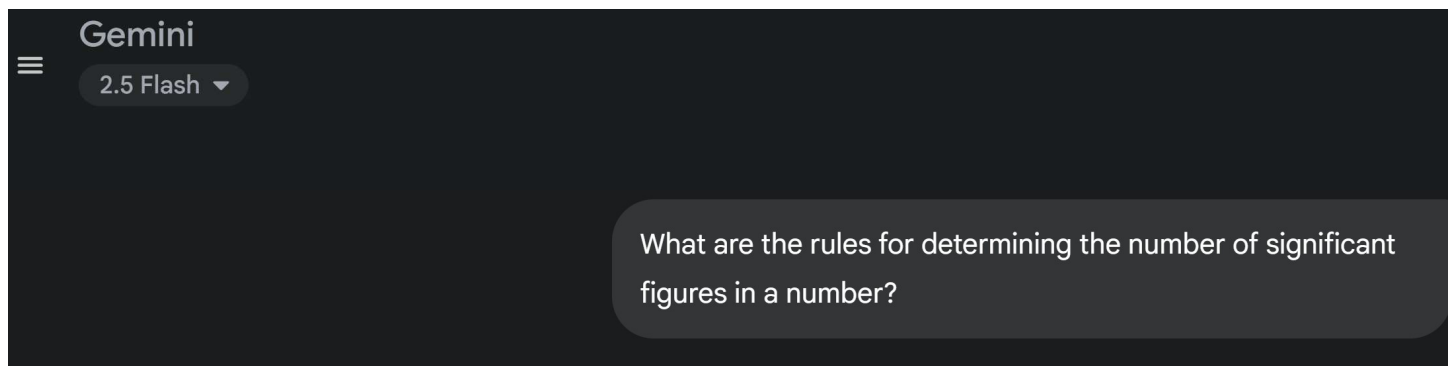
2.1 Linear Equations

2.1 Exercise Set, page 204 (186): 1, 3, 6, 13, 15, 19, 20, 21, 33, 37, 50

2.2 Use a General Strategy to Solve Linear Equations

2.2 Exercise Set, page 221 (203): 1, 8, 15, 27, 30, 32, 34, 40

supplied



- **Non-zero digits are always significant.** For example, the number 456 has three significant figures. 
- **Zeros between non-zero digits are significant.** For instance, the number 101 has three significant figures because the zero is "trapped" between the two ones.  
- **Leading zeros (zeros to the left of the first non-zero digit) are never significant.** They are simply placeholders. For example, the number 0.0075 has only two significant figures (the 7 and the 5).  

- **Trailing zeros (zeros at the end of a number) are significant only if the number contains a decimal point.** For example, 1200 has two significant figures (the 1 and the 2), while 1200. has four significant figures. Writing the number in scientific notation, like 1.20×10^3 , removes this ambiguity by explicitly showing the desired number of significant figures.
- **Exact numbers have an infinite number of significant figures.** These are numbers that are determined by counting (e.g., 12 eggs in a dozen) or by definition (e.g., exactly 100 centimeters in 1 meter).

Now that you have the basic rules, would you like to practice with some examples or learn about how these rules apply to calculations involving addition, subtraction, multiplication, and division?

[Significant Figures - A Fast Review!](#) This video provides a quick review and additional examples on the rules for significant figures.

Definition

The significant figures of a given number are those significant or important digits, which convey the meaning according to its accuracy. For example, 6.658 has four significant digits. These substantial figures provide precision to the numbers. They are also termed as significant digits.

Rules for Significant Figures

- All non-zero digits are significant. 198745 contains six significant digits.
- All zeros that occur between any two non zero digits are significant. For example, 108.0097 contains seven significant digits.
- All zeros that are on the right of a decimal point and also to the left of a non-zero digit is never significant. For example, 0.00798 contained three significant digits.
- All zeros that are on the right of a decimal point are significant, only if, a non-zero digit does not follow them. For example, 20.00 contains four significant digits.
- All the zeros that are on the right of the last non-zero digit, after the decimal point, are significant. For example, 0.0079800 contains five significant digits.
- All the zeros that are on the right of the last non-zero digit are significant if they come from a measurement. For example, 1090 m contains four significant digits.

2.1

Memorize

Solution of an equation

A **solution of an equation** is a value of a variable that makes a true statement when substituted into the equation.

Is $x = 3$ a solution of

$$2x + 1 = 5 ?$$

$$\begin{array}{r} 2(3) + 1 \stackrel{?}{=} 5 \\ 6 + 1 \stackrel{?}{=} 5 \\ 7 \neq 5 \end{array}$$

$$6 + 1 = 7$$

$$7 \neq 5$$

$\therefore x = 3$ is not a solution

HOW TO: Determine whether a number is a solution to an equation

1. Substitute the number in for the variable in the equation.
2. Simplify the expressions on both sides of the equation.
3. Determine whether the resulting equation is true (the left side is equal to the right side).
 - If it is true, the number is a solution.
 - If it is not true, the number is not a solution.

Memorize

Properties of Equality

Subtraction Property of Equality

For any real numbers a , b , and c ,

if $a = b$,
then $a - c = b - c$.

Division Property of Equality

For any numbers a , b , and c , and $c \neq 0$,

if $a = b$,
then $\frac{a}{c} = \frac{b}{c}$.

Addition Property of Equality

For any real numbers a , b , and c ,

if $a = b$,
then $a + c = b + c$.

Multiplication Property of Equality

For any numbers a , b , and c ,

if $a = b$,
then $ac = bc$.

When you add, subtract, multiply, or divide the same quantity from both sides of an equation, you still have equality.

Understand this. Note that we may want to clear fractions first.

HOW TO: Equations with variables and constants on both sides of the equation (beginning strategy)

1. Choose which side will be the “variable” side—the other side will be the “constant” side.
2. Collect the variable terms to the “variable” side of the equation, using the Addition or Subtraction Property of Equality.
3. Collect all the constants to the other side of the equation, using the Addition or Subtraction Property of Equality.
4. Make the coefficient of the variable equal 1, using the Multiplication or Division Property of Equality.
5. Check the solution by substituting it into the original equation.

EXAMPLE 13

$$\left(5\frac{1}{2}x\right) \text{ bad notation}$$

EXAMPLE 13

Solve: $\frac{5}{4}x + 6 = \frac{1}{4}x - 2$.

$\left(\frac{5}{4}x\right)$ bad notation
 $\frac{5x}{4} = \left(\frac{5}{4}\right)x$ good notation

$$\left(\frac{5x}{4}\right)(4) + (6)(4) = \left(\frac{x}{4}\right)(4) - (2)(4)$$

$$5x + 24 = x - 8$$

$$5x - x = -8 - 24$$

$$4x = -32$$

$$x = -8$$

$$\left(\frac{5}{4}\right)(-8) + 6 \stackrel{?}{=} \frac{1}{4}(-8) - 2$$

$$-10 + 6 \stackrel{?}{=} -2 - 2$$

$$-4 = -4 \quad \checkmark$$

solve

$$7x + 14 = 49x + 21$$

$$\frac{7x}{7} + \frac{14}{7} = \frac{49x}{7} + \frac{21}{7}$$

$$x + 2 = 7x + 3$$

$$-1 = 6x$$

$$x = -\frac{1}{6}$$

$$7\left(-\frac{1}{6}\right) + 14 \stackrel{?}{=} 49\left(-\frac{1}{6}\right) + 21$$

$$-\frac{7}{6} + 14 \stackrel{?}{=} -\frac{49}{6} + 21$$

$$-\frac{7}{6} + \frac{(14)(6)}{6} \stackrel{?}{=} -\frac{49}{6} + \frac{2(6)}{6}$$

careless error

$$-\frac{7}{6} + \frac{(14)(6)}{6} \stackrel{?}{=} -\frac{49}{6} + \frac{2(6)}{6}$$

$$-\frac{7}{6} + \frac{84}{6} \stackrel{?}{=} -\frac{49}{6} + \frac{12}{6}$$

$$-\frac{27}{6} \stackrel{?}{=} -\frac{37}{6}$$

I found by copying error

$$7(-\frac{1}{6}) + 14 \stackrel{?}{=} 49(-\frac{1}{6}) + 21$$

$$-\frac{7}{6} + \frac{14(6)}{6} \stackrel{?}{=} -\frac{49}{6} + \frac{(21)(6)}{6}$$

$$-\frac{7}{6} + \frac{84}{6} \stackrel{?}{=} -\frac{49}{6} + \frac{126}{6}$$

$$-\frac{27}{6} = -\frac{27}{6} \quad \checkmark$$

$$\begin{array}{r} 126 \\ -49 \\ \hline 77 \end{array}$$

$$\frac{1}{6} \approx 0.1667$$

$$1/6 = 0.1667$$

$$\text{Check: } 6 * 0.1667 = 1.0002 \approx 1$$

$$\frac{1}{6} = 0.1666...$$

2.2

EXAMPLE 1

How to Solve Linear Equations Using the General Strategy

Solve: $-6(x + 3) = 24$.

$$-\frac{6}{6}(x + 3) = \frac{24}{6}$$

$$-(x + 3) = 4$$

$$-x - 3 = 4$$

$$-x = 4 + 3$$

$$-x = 7$$

$$\text{check: } -6(-7 + 3) \stackrel{?}{=} 24$$

$$-6(-4) \stackrel{?}{=} 24$$

$$24 = 24 \quad \checkmark$$

$$\begin{array}{l}
 -x = 4 + 3 \\
 -x = 7 \\
 \boxed{x = -7}
 \end{array}$$

$$\begin{array}{l}
 -6x - 18 = 24 \\
 -6x = 24 + 18 \\
 -6x = 42 \\
 x = \frac{42}{-6} \\
 \boxed{x = -7}
 \end{array}$$

EXAMPLE 2

Textbook typo

Solve: $(y + 9) = 8$.

Memorize

Conditional equation

An equation that is true for one or more values of the variable and false for all other values of the variable is a conditional equation.

$$\begin{array}{l}
 7x = 14 \\
 \boxed{x = 2}
 \end{array}$$

Definition

Two equations are equivalent if they have the same solution.

When we solve an equation, we are replacing the original equation with a sequence of simpler equivalent equations, until we end up with $x = \text{a number}$.

Identity

An equation that is true for any value of the variable is called an **identity**.

$$\begin{aligned}
 x + x &= 2x \\
 2x &= 2x \\
 2x - 2x &= 2x - 2x \\
 0 &= 0
 \end{aligned}$$

Contradiction

An equation that is false for all values of the variable is called a contradiction.
A contradiction has no solution.

$$\begin{aligned}
 x &= x + 1 \\
 x - x &= x - x + 1 \\
 0 &= 0 + 1 \\
 0 &\neq 1 \\
 \therefore &\text{No solution}
 \end{aligned}$$

Memorize

Type of equation – Solution

Type of equation	What happens when you solve it?	Solution
Conditional Equation	True for one or more values of the variables and false for all other values	One or more values
Identity	True for any value of the variable	All real numbers
Contradiction	False for all values of the variable	No solution

2.2

In the following exercises, solve each linear equation. *check.*

8. $-2(11 - 7x) + 54 = 4$

Classify this equation.

$$\begin{aligned}
 -\frac{2}{2}(11 - 7x) + \frac{54}{-2} &= \frac{4}{-2} \\
 11 - 7x - 27 &= -2 \\
 -7x + (11 - 27) &= -2 \\
 -7x - 16 &= -2 \\
 -7x &= 14 \\
 x &= -2
 \end{aligned}
 \quad \Bigg| \quad
 \begin{aligned}
 -22 + 14x + 54 &= 4 \\
 14x + 32 &= 4 \\
 14x &= -28 \\
 x &= -\frac{28}{14} \\
 x &= -2
 \end{aligned}$$

$$\begin{array}{l|l}
 -7x + 16 = -2 & x = -\frac{28}{14} \\
 7x + 16 = 2 & \boxed{x = -2} \\
 7x = 2 - 16 & \\
 7x = -14 & \\
 x = -\frac{14}{7} & \\
 \boxed{x = -2} &
 \end{array}$$

conditional

Non-linear equation $(x-4)(x+5) = 0$

$$\begin{array}{l}
 x-4=0 \text{ or } x+5=0 \\
 \boxed{x=4 \text{ or } x=-5} \\
 (x-4)(x+5) = x^2 - 4x + 5x - 20 \\
 = x^2 + x - 20 \\
 \text{quadratic function}
 \end{array}$$

In the following exercises, classify each equation as a conditional equation, an identity, or a contradiction and then state the solution.

31. $9(a-4) + 3(2a+5) = 7(3a-4) - 6a + 7$



Analysis ▼

This equation is a **conditional equation**. The solution is $a = 6$.

Solving the Equation

To determine the type of equation and find the solution, we need to simplify both sides of the equation:

$$9(a - 4) + 3(2a + 5) = 7(3a - 4) - 6a + 7$$

Left-hand side (LHS):

Distribute the 9 and the 3:

$$9a - 36 + 6a + 15$$

Combine like terms ($9a + 6a$ and $-36 + 15$):

$$15a - 21$$

Right-hand side (RHS):

Distribute the 7:

$$21a - 28 - 6a + 7$$

Combine like terms ($21a - 6a$ and $-28 + 7$):

$$15a - 21$$

Now, set the simplified sides equal to each other:

$$15a - 21 = 15a - 21$$

When the simplified left side is identical to the simplified right side, the equation is an **identity**. An identity is true for all values of the variable, so the solution is all real numbers.

Summary of Equation Types

- **Conditional Equation:** An equation that is true for **specific values** of the variable. This is what we found in this problem. For example, $x + 1 = 5$ is only true when $x = 4$.
- **Identity:** An equation that is true for **all real numbers**. The simplified forms of both sides are identical. For example, $2x + 4 = 2(x + 2)$ is an identity.
- **Contradiction:** An equation that is **never true** for any value of the variable. The simplified forms of both sides are equal to different constants. For example, $x + 1 = x + 2$ is a contradiction.

Wait a minute! How can this be both conditional with $a = 6$ and an identity?

You've brought up an excellent point, and it's a common area of confusion. My previous response contained a mistake. My apologies for that. The equation you provided is an **identity**, not a conditional equation. Let's break down why.

Again, my apologies for the initial misclassification. It's a great lesson in the importance of double-checking your work, especially in mathematics.