09-12-25 MTH 111

1.2 Integers
  1.2 Exercise Set, page 69 (51): 1, 2, 6, 18, 25, 32, 39, 40, 48
1.4 Decimals
  1.4 Exercise Set, page 112 (94): 1, 2, 4, 5, 9, 10, 18, 19, 25, 31, 33, 42, 44

1.5 Exponents and Scientific Notation
  1.5 Exercise Set, page 142 (124): 1, 3, 6, 8, 11, 18, 20, 22, 26, 28, 30, 32, 34, 36, 38, 41
1.6 Roots and Radicals
  1.6 Exercise Set, page 163 (145): 1, 3, 12, 14, 15, 17,19

I will provide supplementary material about identifying significant digits. (later)

Exam 1, Friday, 09/19/25, 1.1 - 1.6

1.5
Memorize

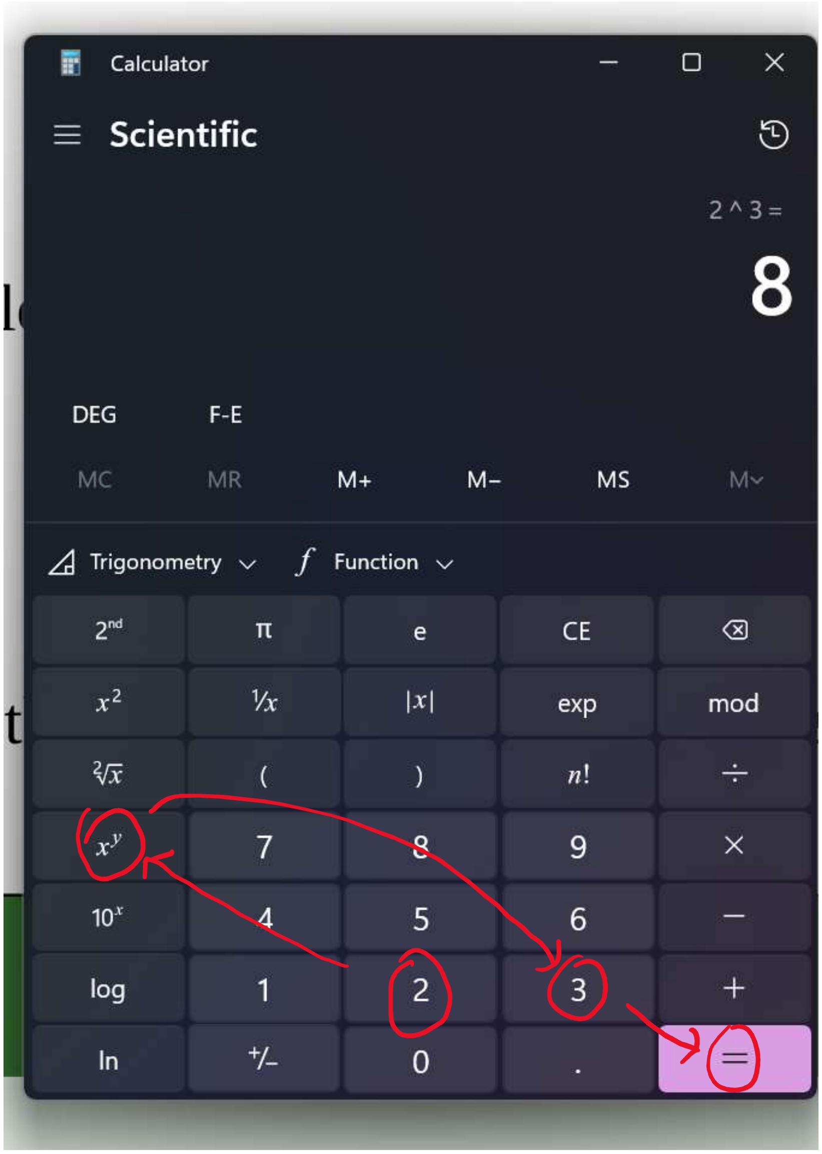
Exponential Notation (Power)

$$\underset{\text{base}}{a}^{m \leftarrow \text{exponent}}$$

a^m means multiply m factors of a

$$a^m = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{m\ factors}$$

a raised to the mth power



2 ^ 3 enter → 8

Memorize

Product Property for Exponents

If a is a real number, and m and n are counting numbers, then

$a^m \cdot a^n = a^{m+n}$

Let $a = 2$

$m = 3$

$n = 2$

$$2^3 \cdot 2^2 = (2\cdot 2\cdot 2)(2\cdot 2)$$
$$= 2\cdot 2\cdot 2\cdot 2\cdot 2$$
$$= 2^5 = 2^{3+2}$$

Memorize

Quotient Property for Exponents

If a is a real number, $a \neq 0$, and m and n are whole numbers, then

$\dfrac{a^m}{a^n} = a^{m-n}, m > n$ and $\dfrac{a^m}{a^n} = \dfrac{1}{a^{n-m}}, n > m$

$a = 2$
$m = 4$
$n = 2$

$$\frac{2^4}{2^2} = \frac{2\cdot 2\cdot \cancel{2}\cdot \cancel{2}}{\cancel{2}\cdot \cancel{2}} = 2\cdot 2 = 2^2 = 2^{4-2}$$

$$\frac{2^3}{2^3} = \frac{8}{8} = 1$$

By quotient rule: $\dfrac{2^3}{2^3} = 2^{3-3} = 2^0$

This makes no sense.

However, to keep our beautiful rule,

define $2^0 = 1$

Zero Exponent

If a is a non-zero number, then $a^0 = 1$.

Any nonzero number raised to the zero power is 1

TI-nspire

0^0	undef

$$\frac{2^3}{2^5} = \frac{\cancel{2}\cdot\cancel{2}\cdot\cancel{2}}{2\cdot 2\cdot\cancel{2}\cdot\cancel{2}\cdot\cancel{2}} = \frac{1}{2\cdot 2} = \frac{1}{2^2}$$

by quotient rule: $\frac{2^3}{2^5} = 2^{3-5} = 2^{-2}$, which makes no sense

To keep our rule, define $2^{-2} = \frac{1}{2^2}$

generalize $\boxed{a^{-n} = \frac{1}{a^n}}$

Memorize

Negative Exponent

If n is an integer and $a \neq 0$, then $a^{-n} = \frac{1}{a^n}$.

$$2^{-(-3)} = \frac{1}{2^{-3}}$$

$$2^3 = \frac{1}{\frac{1}{2^3}}$$

$$2^3 = 2^3$$

$$\frac{2^3}{2^2} = \frac{2\cdot\cancel{2}\cdot\cancel{2}}{\cancel{2}\cdot\cancel{2}} = 2$$

by rule: $\frac{2^3}{2^2} = 2^{3-2} = 2^1$

$\therefore$ define $2^1 = 2$

$\therefore$ define $2^1 = 2$

generalize $\boxed{a^1 = a}$

extra $2^{\frac{1}{2}} \cdot 2^{\frac{1}{2}}$

$2^{\frac{1}{2}} \cdot 2^{\frac{1}{2}}$ 2

$\sqrt{2}\sqrt{2} = 2$

To keep our product rule

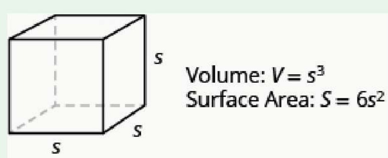
define $2^{\frac{1}{2}} = \sqrt{2}$

generalize $a^{\frac{1}{n}} = \sqrt[n]{a}$

Memorize

Volume and Surface Area of a Cube

For any cube with sides of length s,



Supplied

Volume and Surface Area of a Sphere

For a sphere with radius r:

Memorize

Scientific Notation

A number is expressed in scientific notation when it is of the form $a \times 10^n$ where $1 \leq a < 10$ and n is an integer

$781 = 7.81 \times 10^2$

$1 \leq 7.81 < 10$

$0.0035 = 3.5 \times 10^{-3}$

$1 \leq 3.5 < 10$

Memorize

HOW TO: Convert from decimal notation to scientific notation

1. Move the decimal point so that the first factor is greater than or equal to 1 but less than 10.
2. Count the number of decimal places, n, that the decimal point was moved.
3. Write the number as a product with a power of 10.
   If the original number is:

Memorize

HOW TO: Convert scientific notation to decimal form.

To convert scientific notation to decimal form:

1. Determine the exponent, n, on the factor 10.
2. Move the decimal n places, adding zeros if needed.
    - If the exponent is positive, move the decimal point n places to the right.
    - If the exponent is negative, move the decimal point $|n|$ places to the left.
3. Check.

Use scientific notation and rounding to single digits to estimate a calculation.
Then, do the calculation on your calculator and compare the orders of magnitude,
That is, the powers of 10.

$$\frac{(35.72)(0.04)}{(0.0007)(6)}$$

35.72*0.04/(0.0007*6)=340.1905

$\approx \boxed{3\times10^{2}}$

$$= \frac{(3.572\times10^{1})(4\times10^{-2})}{(7\times10^{-4})(6\times10^{0})}$$

$$\approx \frac{(4\times10^{1})(4\times10^{-2})}{(7\times10^{-4})(6\times10^{0})}$$

$$= \frac{(4)(\cancel{4}^{\,2})\times10^{1-2}}{(7)(\cancel{6}_{\,3})\times10^{-4+0}}$$

$$= \left(\frac{8}{21}\right)\frac{\times10^{-1}}{10^{-4}}$$

$$\approx (.4)\times10^{-1-(-4)}$$

$$\begin{array}{r} .38 \\ 21\,\overline{)\,8.00} \\ 6\,3 \\ \hline 1\,70 \\ 1\,68 \\ \hline 2 \end{array}$$

$\frac{168}{2}$

$= 0.4 \times 10^3$

$= \boxed{4 \times 10^2} \approx \boxed{3 \times 10^2}$

same order of magnitude

$(0.4)(1000) = \left(\frac{4}{10}\right)(1000)$

$= (4)\left(\frac{1000}{10}\right) = 4(100) = 400$

$= 4 \times 10^2$

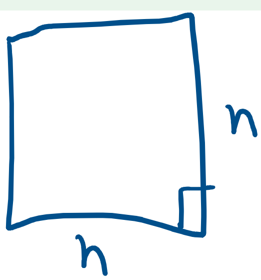
0.4×10^3

$= (0.4)(10) \times \frac{10^3}{10}$

1.6
Memorize

Square of a Number

If $n^2 = m$, then *m* is the **square** of *n*.



area of square with side n is n^2

Memorize

Square Root of a Number

If $n^2 = m$, then n is a **square root** of m.

$5^2 = 25 \Leftrightarrow \sqrt{25} = 5$

Solve $x^2 = 25$

$x = \pm\sqrt{25}$

$x = \pm 5$

Memorize

Square Root Notation

$\sqrt{m}$ is read "the square root of *m*"

radical sign $\longrightarrow \sqrt{m} \longleftarrow$ radicand

If $m = n^2$, then $\sqrt{m} = n$, for $n \geq 0$.

The square root of *m*, $\sqrt{m}$, is the positive number whose square is *m*.

supplied

n^{th} Root of a Number

If $b^n = a$, then b is an n^{th} root of a.

The principal n^{th} root of a is written $\sqrt[n]{a}$.

n is called the index of the radical.

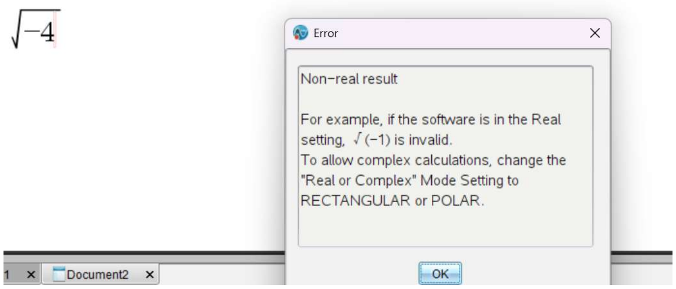
Memorize

Properties of $\sqrt[n]{a}$

When n is an even number and

- $a \geq 0$, then $\sqrt[n]{a}$ is a real number.
- $a < 0$, then $\sqrt[n]{a}$ is not a real number.

When n is an odd number, $\sqrt[n]{a}$ is a real number for all values of a.



$\sqrt{-4} = 2i$, where $i^2 = -1$

i^2 -1

Estimate a square root by finding an integer n such that $n^2 < radicand < (n+1)^2$.
Estimate

$\sqrt{30}$

$4^2 = 16 < 30$

$5^2 = 25 < 30$

$6^2 = 36 > 30$

$\therefore \boxed{5 < \sqrt{30} < 6}$

Sqrt(30)=5.47722557505166

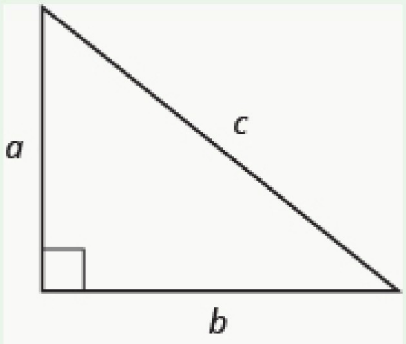
$5 < 5.477 < 6$

The Pythagorean Theorem

In any right triangle ΔABC,

$a^2 + b^2 = c^2$

where c is the length of the hypotenuse a and b are the lengths of the legs.



The converse is also true.
If a triangle with sides a, b, and c satisfies $a^2 + b^2 = c^2$,
Then the triangle is a right triangle, and c = the hyptenuse and
a and b are the legs.

Simplify radicals

$\sqrt{25} = 5$

$\sqrt{12}$

$3^2 = 9 < 12$

$4^2 = 16 > 12$

$3 < \sqrt{12} < 4$

memorize $\sqrt{ab} = \sqrt{a}\sqrt{b}$

$\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4}\sqrt{3}$

$= \boxed{2\sqrt{3}}$

$\sqrt{17}$	already simplified	$\sqrt{17}$
$\sqrt{12}$	simplified	$2 \cdot \sqrt{3}$

$\sqrt{12}$ $\xrightarrow{\text{simplified}}$ $2\cdot\sqrt{3}$

simplify $\sqrt{24}$

24

6 4

3 2 2 2 prime number

$24 = 2^3 \cdot 3 = (2\cdot 2\cdot 2)(3) = (2\cdot 2)(2\cdot 3)$

$= (2^2)(2\cdot 3)$

$\sqrt{24} = \sqrt{2^2}\sqrt{6}$

$= \sqrt{4}\sqrt{6}$

$= \boxed{2\sqrt{6}}$

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① Convert $8\underbrace{46}_{2}.2$ to scientific notation

8.462×10^{2}

② calculate manually $\dfrac{(400)(60)}{(0.8)(3)}$

write answer in scientific notation

$$\frac{(4\times 10^{2})(6\times 10^{1})}{(8\times 10^{-1})(3\times 10^{0})}$$

$2+1$

$$(8\times10^{-1})(3\times10^{0})$$

$$= \frac{(\cancel{4})(\cancel{6})^{2}}{(\cancel{8})_{2}(\cancel{3})} \times \frac{10^{2+1}}{10^{-1+0}}$$

$$= \left(\frac{2}{2}\right) \times \frac{10^{3}}{10^{-1}}$$

$$= 1 \times 10^{3-(-1)}$$

$$= 1 \times 10^{3+1}$$

$$\boxed{= 1 \times 10^{4}}$$

simplify $\sqrt{120}$

find perfect square factor

use $\sqrt{ab} = \sqrt{a}\sqrt{b}$

Then estimate $\sqrt{120}$ by finding integer n such that

$$n < \sqrt{120} < n+1$$

$$n^2 < 120 < (n+1)^2$$

Then use calculator to estimate $\sqrt{120}$, rounded to nearest hundredth

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$\sqrt{120}$

120
/\

$120 = 2^3 \cdot 3 \cdot 5$

$= (2^2)(2 \cdot 3 \cdot 5)$

$\sqrt{120} = \sqrt{2^2}\sqrt{2 \cdot 3 \cdot 5}$

$\boxed{\sqrt{120} = 2\sqrt{30}}$

$10^2 = 100 < 120$

$11^2 = 121 > 120$

$10 < \sqrt{120} < 11$

$5^2 = 25 < 30$

$6^2 = 36 > 30$

$5 < \sqrt{30} < 6$

$2 \cdot 5 < 2\sqrt{30} < 2 \cdot 6$

$10 < 2\sqrt{30} < 12$

too big

1.5 not integer

0 integer

Integer

$$(3)(-5) = -15$$

$$(-3)(-5) = 15$$