04-20-26 MDE 10

8.3 Adding Real Numbers
8.3 Exercise Set, page 581: 1,11, 27, 57, 59, 65, 67, 79, 87

8.4 Subtracting Real Numbers
8.4 Exercise Set, page 590: 3, 13,19, 71

8.5 Multiplying and Dividing Real Numbers
8.5 Exercise Set, page 605: 5, 9, 11, 25, 35, 39, 59, 91, 105, 113, 125
8.6 Properties of Real Numbers
8.6 Exercise Set, page 615: 1, 5, 9, 15, 17,27, 41, 63

8.7 Simplifying Expressions
8.7 Exercise Set, page 623: 1, 3, 7, 9, 19, 41, 59, 77, 81

10 textbook sections
5 class meetings before final exam
2-3 textbook sections per class meeting

8.4: 109

Without calculating, determine whether each answer is positive or negative. Then use a calculator to find the exact difference.

109. 56,875 − 87,262

56875-87262=-30387

$-10+(-12)=$

8.3

Each calculation below is incorrect. Find the error and correct it. See the Concept Check in this section.

109. $-10 + (-12) \stackrel{?}{=} -120$

$-10 - 12 \stackrel{?}{=} -120$

$-22 \neq -120$

8.5

Memorize

Multiplying Real Numbers

1. The product of two numbers with the *same* sign is a positive number.

2. The product of two numbers with *different* signs is a negative number.

Memorize

Products Involving Zero

If b is a real number, then $b \cdot 0 = 0$. Also $0 \cdot b = 0$.

Memorize

Helpful Hint

Have you noticed a pattern when multiplying signed numbers?

If we let $(-)$ represent a negative number and $(+)$ represent a positive number, then

$$(-)(-) = (+)$$
$$(-)(-)(-) = (-)$$
$$(-)(-)(-)(-) = (+)$$
$$(-)(-)(-)(-)(-) = (-)$$

The product of an even number of negative numbers is a positive result.

The product of an odd number of negative numbers is a negative result.

Memorize

Reciprocal or Multiplicative Inverse

Two numbers whose product is 1 are called **reciprocals** or **multiplicative inverses** of each other.

inverses of each other.

$(3)\left(\frac{1}{3}\right) = 1$

1 is the multiplicative identity

$a \cdot 1 = a$ for any a

$3 + (-3) = 0$

0 is the additive identity

$a + 0 = a$ for any a

Memorize

Quotients Involving Zero

The number 0 does not have a reciprocal.

Division Involving Zero

If a is a nonzero number, then $\frac{0}{a} = 0$ and $\frac{a}{0}$ is undefined.

$\frac{0}{a} = 0\left(\frac{1}{a}\right) = 0$

memorize

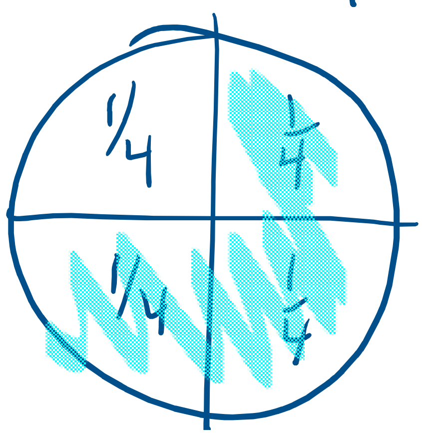
Quotient of Two Real Numbers

If a and b are real numbers and b is not 0, then

$$a \div b = \frac{a}{b} = a \cdot \frac{1}{b}$$

$\frac{3}{4} = 3\left(\frac{1}{4}\right)$

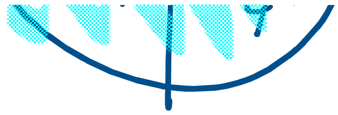
$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$



$\frac{3}{4}$

$$\begin{array}{r} 0.75 \\ 4\overline{)\,3.00} \\ \underline{2\,8} \\ 20 \end{array}$$

$\frac{3}{4} = 0.75$



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$\frac{3}{4} = 0.75$

Memorize

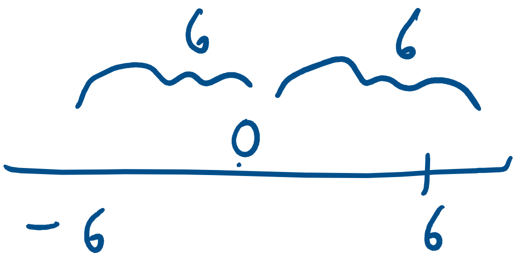
Dividing Real Numbers

1. The quotient of two numbers with the *same* sign is a positive number.

2. The quotient of two numbers with *different* signs is a negative number.

Memorize

If a and b are real numbers, and $b \neq 0$, then $\frac{a}{-b} = \frac{-a}{b} = -\frac{a}{b}$.



$|-6| = 6$

$|6| = 6$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$6 \geq 0 \Rightarrow |6| = 6$

$-6 < 0 \Rightarrow |-6| = -(-6) = 6$

Memorize

Definition: A solution to an equation is a value of the variable, or variables, that make the equation true.

$x^2 + 3 = y$

I guess that $x=1, y=2$ is a solution.

Check: $1^2 + 3 \stackrel{?}{=} 2$

$1 + 3 \stackrel{?}{=} 2$

$4 \neq 2$

$x=1, y=2$ is <u>not</u> a solution

Guess $x=1, y=4$

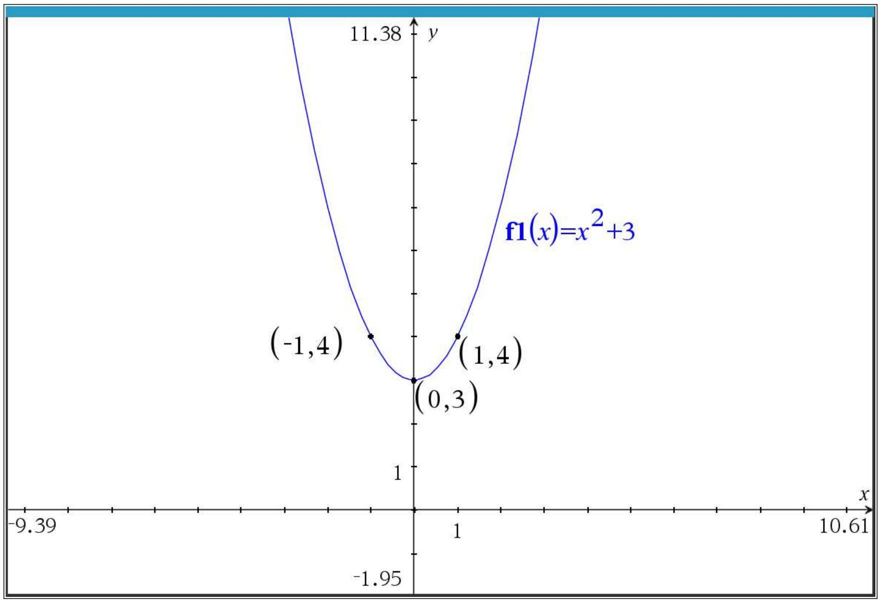
$1^2 + 3 \stackrel{?}{=} 4$

$1 + 3 \stackrel{?}{=} 4$

$4 = 4$

$\therefore (x,y) = (1,4)$ <u>is</u> a solution

Preview of analytic geometry each ordered pair of numbers is one solution of $y = x^2 + 2$



8.6
Memorize

Commutative Properties

Addition:	$a + b = b + a$
Multiplication:	$a \cdot b = b \cdot a$

$3 - 2 = 1$

$2 - 3 = -1$

$1 \neq -1$

$\therefore$ subtraction is <u>not</u> commutative

$2 \div 3 = \frac{2}{3}$

$3 \div 2 = \frac{3}{2}$

$\frac{2}{3} \neq \frac{3}{2}$

$3 \neq 2$

$\therefore$ division is not commutative

$3-3=0$
$3-3=0$ } this example does not prove that subtraction is commutative

Memorize

Associative Properties

Addition:	$(a+b)+c=a+(b+c)$
Multiplication:	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$

$(a-b)-c \stackrel{?}{=} a-(b-c)$

Let $a=3$
$b=2$
$c=1$

$(3-2)-1 \stackrel{?}{=} 3-(2-1)$

$1-1 \stackrel{?}{=} 3-(1)$

$0 \neq 2$

$\therefore$ subtraction is not associative

$(\ \ \ )\ \ \ = a-(b-c)$

$$(a-b)-c = a-(b-c)$$

$$\Leftrightarrow \quad a-b-c = a-b+c$$

$$\Leftrightarrow \quad -b-c = -b+c$$

$$\Leftrightarrow \quad -c = c$$

$$\Leftrightarrow \quad 0 = c+c$$

$$0 = 2c$$

$$\boxed{0 = c}$$

$$(a \div b) \div c \overset{?}{=} a \div (b \div c)$$

$$\frac{\frac{a}{b}}{c} \overset{?}{=} \frac{a}{\frac{b}{c}}$$

$$\frac{a}{b} \cdot \frac{c}{1} = \left(\frac{a}{1}\right)\left(\frac{c}{b}\right)$$

$$\frac{ac}{b} = \frac{ac}{b}$$

Memorize

Distributive Property of Multiplication Over Addition

$$a(b + c) = ab + ac$$

$\longrightarrow$ multiply out (expand)

$\longleftarrow$ factor

Memorize

Identities for Addition and Multiplication

0 is the identity element for addition.

$a + 0 = a$ and $0 + a = a$

1 is the identity element for multiplication.

$a \cdot 1 = a$ and $1 \cdot a = a$

Memorize

Additive or Multiplicative Inverses

The numbers a and $-a$ are additive inverses or opposites of each other because their sum is 0; that is,

$a + (-a) = 0$

The numbers b and $\frac{1}{b}$ (for $b \neq 0$) are reciprocals or multiplicative inverses of each other because their product is 1; that is,

$b \cdot \frac{1}{b} = 1$